STRENGTH AND DETERMINATION

OF THE

DIMENSIONS

OF

STRUCTURES OF IRON AND STEEL

WITH

REFERENCE TO THE LATEST INVESTIGATIONS.

AN ELEMENTARY APPENDIX TO ALL TEXT-BOOKS UPON IRON AND STEEL CONSTRUCTION.

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The present English translation of my „Festigkeit und Dimensionenberechnung" has been kindly made by Prof. Du Bois at my special request, and by the courtesy of the Publishers I receive a liberal copyright therefor. No other English translation has been or will be authorized by me.

Stuttgart, 11. March 77. Jacob J Weyrauch

AUTHOR'S PREFACE TO THE AMERICAN EDITION.

THE methods of calculation of the forces which act upon the various members of our bridges and other structures have, within the last ten years, owing to the united labor of European and American engineers, gained most remarkably in clearness and reliability. These advances can, however, attain their greatest value, only when the question as to what forces these members can sustain with the desired degree of security is satisfactorily disposed of.

That the method of dimensioning thus far in use is an entirely arbitrary one, cannot be denied. In spite of numerous attacks no defence of it has ever been heard. What has for the last hundred years justified the assumption that a piece which has once successfully resisted a certain stress, must necessarily resist equally well an indefinite number of repetitions of that stress? How can it be held that it is a matter of indifference whether a piece is subjected always to the same constant load, or is alternately loaded and then unloaded, or is even subjected to alternate strains of tension and compression? Every layman knows that he can more readily break a piece by bending to and fro than by a steady pull, even though the force exerted in each case be the same.

By assuming the strength, which is *not* constant, as nevertheless constant for every member of a construction, the degree of safety of the different members varies. The least safety of any place in the structure is, however, the measure of the security of the whole. If one member gives way, it is a matter of little moment whether, in falling, the other members hang together or not, and the structure comes to the ground in two or more pieces.

AUTHOR'S PREFACE TO THE AMERICAN EDITION.

The admirable investigations made during the last eighteen years at the instance of the Prussian Government, have led to results in complete accord with practical sense. The method by which, even before the publication of these results, in the construction of the bridge over the Rhine at Mayence, the strength of the web members was estimated, appears to have been, in all its esssentials, correct. The most important result of the investigations alluded to, however, is "Wöhler's law;" a principle which is, indeed, self-evident, and upon which, in future, every rational method of dimensioning must be based. Immediately after the publication of Wöhler's investigations, the results of which were further confirmed by Spangenberg, new methods of dimensioning were proposed in Germany and Austria. These methods are reviewed and criticised in the Appendix to the present work. These proposed methods are, all of them, not sufficiently developed, and are all, moreover, too closely fitted to the *numerical* results of Wöhler.

The present little work gives a systematic presentation of a new method of dimensioning, based upon two formulæ deduced by Professor Launhardt and by the Author, respectively. It will be noticed that this method, which leaves as to simplicity nothing to be desired, gives considerable economy of material as well as increased security, while the ordinary methods of statical calculation, in general use, remain unaffected by it.

As the resistance of riveted constructions depends directly upon the quality of the rivet connections, I have gived special attention to this hitherto somewhat neglected subject. Without denying the numerous advantages of bolt connections, European engineers use rivets even in frame work, almost exclusively.

In order to make the work serviceable to the practical engineer, I have subjected the numerous experiments upon the strength properties of Iron and Steel recently made in various countries to careful comparison and scrutiny, and

thus given a concise presentation of the present state of knowledge in this respect. Every thing is thus given which, after the completion of the *statical* calculation, may be desired.

Finally, I must take occasion to express my best thanks to Professor Du Bois for his kind compliance with my wishes in the preparation of this Translation, which will, I hope, meet with as favorable a reception from the Profession in America as has already been so justly accorded to the Translator's "Elements of Graphical Statics."

JACOB J. WEYRAUCH.

STUTTGART, *March*, 1877.

TRANSLATOR'S PREFACE.

THE present translation is made with the consent and in accordance with the expressed wish of the author, who, by the courtesy of the publishers, will receive an unsolicited copyright upon its sale.

It has not been thought necessary, in view of the present acceptance of the metric system in this country, to give only the reduced values of dimensions. In the present case, at least, no difficulty can be experienced by any one, the most commonly occurring measure being *kilograms per square centimetre;* and since 700 kilograms per square centimetre are equivalent almost exactly to 10,000 lbs. per square inch, we have only in any case to multiply by 100 and divide by 7 in order to obtain pounds per square inch. The English equivalents for other values are, in general, bracketed alongside of the measures to which they apply, and upon the last page of the work will be found also reduction tables, which, however, it is hoped, will be entirely unnecessary.

As to the merits of this little work, and its peculiar interest at the present time, but little need be said. The author's preface and the text will speak for themselves, and both will, we think, be found to the point. No one interested in constructions involving the use of iron or steel can afford to ignore any longer the results here set forth. We have to do here, not with the results of theoretical reason-

ing, itself based perhaps upon assumptions more or less questionable, but with legitimate deductions from the results of varied and careful experiment. It is but simple justice to American engineering to state here, that its best representatives have already long accepted and made use of these results. The author has paid to American practice in this respect a well-merited tribute, and there are, we trust, few who do not now recognize in the choice of their unit strains a difference between the effect of simple strains of tension and compression alone, of repeated tension *or* compression, and of alternate tension *and* compression, occurring in the different members of a bridge-truss. Still, even to such, we think, the following systematic presentation of a general method, applicable to all cases, will be found of value; while we are forced to think that, to a large majority of those interested in such constructions, much that is here given will be as new as it is valuable and even indispensable. More attention to just such facts as are here set forth and worked into a general method of "dimensioning"—facts which have long been at disposal, but never before properly set forth in a shape to meet the daily wants of the practising engineer and constructor—would make such sad disasters as that at *Ashtabula* impossible, and go far to restore public confidence in a class of constructions whose only too frequent failures of late years render such a restoration of confidence most desirable.

It will be seen that in the list of distinguished experimenters who have contributed to the mass of accurate knowledge which renders such a work as the present possible, American engineers occupy a prominent place, thus justifying their well-earned reputation as among the fore-

most *constructors* of the world, and worthy representatives of those predecessors whose achievements with wood alone first made American engineering a recognized fact abroad. The intelligent constructor is not apt to lose sight of the conditions imposed by the material with which he works; and the recognition by the author of American practice; which antedates anything of the kind abroad, imperfect though it may be in the light of present knowledge, only goes to show, if proof now were needed, that the so-called "practical engineer" of the present day is guided no longer by "rules of thumb" alone, but is fully alive to the necessity of an accurate knowledge of the materials with which he works, guided by an intelligent comprehension of the principles which should regulate their use.

The author has taken occasion to call in question certain recent experiments made in this country—whether with reason or not, we leave the reader to decide. For this purpose, and in order that both sides might be fairly presented, Prof. Thurston has, by request, kindly furnished an appendix treating of the strictures referred to in the text. A *critique* by Prof. Kick,* of the Institute of Technology of Prague, upon Prof. Thurston's investigations, which called in question more especially the accuracy of the "autographic recording testing-machine," seems to have been the basis of the author's objections. An answer to this *critique* by Prof. Thurston will be found in the Transactions of the American Society of Civil Engineers, Dec. 31, 1875.

In the appendix by the author will be found a review of the various methods thus far proposed for the "dimension-

* Dingler's "Poly. Journal," Bd. 218, Heft. 3.

ing" of parts, given at such length that these methods may also be used if desired, while their relative merits, as well as the merits of the method here advocated, are well brought out.

The acceptance which this little work has met abroad, and the practical significance of its contents, are sufficient excuse for offering it to the profession in America. It will in any case be found an acceptable "appendix to all text-books upon iron and steel construction."

BETHLEHEM, March 17, 1877.

PREFACE.

In recent times, most admirable experiments have been made upon the properties of iron and steel in Germany, England, Sweden, and America. In the present brochure, it has been attempted to present the tangible results and consequences of these experiments, as free from detail as possible, but to such an extent that the practical engineer may be put in possession of the results thus far obtained. Numerous foot references to the literature of the subject point out the sources of more detailed information upon special topics.

The experiments alluded to have proved, among other things, what is by this time all but universally admitted, that heretofore the method of dimensioning iron and steel constructions has been entirely incorrect, and that the safety of the structures thus proportioned, in spite of liberal expenditure of material, is considerably less than that relied upon (Chaps. 3, 13, 29).

Various methods have been proposed for the attainment of better results, one of which has even been adopted by the Bavarian government. A short presentation of these methods in the Appendix will show that of them, that due to Launhardt deserves the preference. This method is so clear, and open to so few objections, that only its limited applicability is to be regretted. The strength relations for alternate stresses of tension and compression are, however, not included by the formula of Launhardt. Such a formula is deduced in the present treatise. With this lack supplied, all the necessary data for a simple and rational method of dimensioning are at hand. It is to be hoped we shall not delay its acceptance until still more bridges come to grief.

One of the chief reasons why thus far *none* of the new methods

have met with better favor, is that none of them have been sufficiently developed. It has not been possible to proportion a bridge by any one of these methods completely, without instituting special discussions. For such discussions the practising engineer has, in general, little time. A systematic presentation is therefore imperative.

The systematic presentation, complete in itself, here given, includes also the hitherto neglected cases of construction-pieces submitted to shear. Special and much needed attention has been given to rivet connections. Although here also the present condition of theory finds full recognition, I do not consider that it has been at the expense of simplicity of application.

The customary methods of statical calculation remain unaffected by the new method of dimensioning. Those who prefer the graphical methods for such calculations, will find in the present brochure every thing which remains necessary to be done after the preparation of the strain-sheet, in the calculation of a structure. By the previous rude methods of dimensioning, very exact *statical* calculations are rendered rather useless.

In the setting forth of the new formulæ, Wöhler's law has of course been premised. The special experimental values of Wöhler are, however, to be used with caution, and scarcely more weight can be laid upon them relatively than, for example, upon the results of Rondelet or Brunel or others, with regard to the previous methods. The general formulæ are, however, affected as little by such new results as, in the time of Brunel, the early *method* of procedure was by new and later series of experiments (Chaps. 2, 29).

In the values for the allowable stress per square centimetre, we have had special reference to bridges and other constructions from which unlimited life is expected. Study of strength properties and practical experience will furnish sufficient grounds for the proper selection of the coefficient of safety in other cases.

Notwithstanding the simplicity of the contents of the present work, I have endeavored to give with care, perhaps with rather too anxious care, credit, where I have borrowed from others that which has not long been common property, and to indicate authorities and

sources of information. Better thus, than to imitate those who preserve a discreet silence upon such matters, and concerning whose works we do well to take for granted that *all* is compilation.

In works upon iron and steel constructions, there is generally but little to be found upon the strength properties of the materials themselves, and for reasons already alluded to the new methods of dimensioning find no place. I may therefore hope, in this little work, which may serve as an appendix to every text-book, to have supplied a real want.

THE AUTHOR.

STUTTGART, 1876.

GENERAL PROPERTIES.

DETERMINATION OF DIMENSIONS.

THE method of determination of the proper dimensions of the various component parts of constructions of iron and steel has, until recently, been as follows: Having determined by the statical calculation the greatest stress, *max B*, which can ever act upon the part in question, we divide this stress by the assumed allowable stress, b, per unit of area, and thus obtain the required area of cross-section,

$$F = \frac{max\ B}{b} \qquad (1)$$

which the part in question must have in order to safely resist the given stress.

In general the value of b is taken invariable, no matter whether the stress is caused by a constant or "*dead* load," or whether the load, and, therefore, the corresponding stress, undergoes sudden changes, as in the case of bridges, rails, axles, etc. In Prussia, for example, the value of b for iron is almost universally assumed at 730 kil. per square centimetre (about 10,400 lbs. per sq. inch), and this value is assumed indifferently for tension, compression, and shear.

* These remarks do not apply to present American practice. Different values of b are taken for tension, compression, etc., though constant for each.—TRANS.

Gerber, in the calculation of the Mainz bridge, was one of the first to depart from the above practice. He assumed for each member of the construction a different value of b, taking it less as the proportion of the stress due to moving load to that due to the dead load increased.*

When a member is submitted also to alternate tension and compression, formula (1) was still considered as holding good, the *absolute* greatest value being taken for $max\ B$. The American practice has, in such case, been a more rational one. According to it, if $max\ B$ and $max\ B'$ are the greatest strains of opposite character, then

$$F = \frac{max\ B + max\ B'}{b} \qquad (2)$$

where the usual invariable value of b, as above, is, however, assumed. Numberless axle failures, boiler explosions, bridge failures, etc., admonish those interested to search after the causes of these occurrences. Such occurrences are the more noticeable, inasmuch as a coefficient of safety is always used which, it would appear, ought to exclude every danger. The question, therefore, presses hard upon us, whether our iron bridges in general really possess the durability assigned to them. We have, as yet, no experience in this direction to appeal to, since the application of iron in bridge construction reaches back hardly a hundred years. In the year 1874 the Society "Deutscher Architekten- und Ingenieurvereine" determined to seek, by systematic and similar observations, a solution "in order that those engineers engaged

* Ztschr. d. Vereins dtsch. Ingenieure, 1865, p. 463. A similar practice was later followed by Griffen and Clark in America. Compare Ztschr. d. Hannöv. Arch. u. Ing.-Vereins, 1876, p. 94.

in the maintenance of iron structures may no longer be confronted with disasters."

These observations are of the greatest importance, but can, of course, hardly give decisive results for decades to come. Meanwhile we should do well to consider somewhat more closely those observations *at present* available. Let us ask whether our customary method of determining the dimensions of the various members of iron and steel structures is entirely free from criticism. This question must unfortunately be answered in the negative. Admitting this, we shall indeed do well by continuing to experiment and observe, but at the same time we should consign that which has finally been recognized as false as speedily as possible to oblivion.

We shall, therefore, find our best guide for *further* observation in the consideration of those facts placed at our disposal by the theory and experience of the *present day*. "In order to see aright, we must know what to look for," may apply here also, if it may be permitted to quote Schelling in connection with iron bridges.

CHAPTER I.

WÖHLER'S LAW.

THE experiments upon which the old method of calculation of dimensions is founded, have been made within about the last hundred years by Perronet, Poleni, Telford, Brunel, and many others.* Many of these were made with all desirable care, and are by no means valueless now; but they were, however, made from a one-sided standpoint. It was assumed that a body which could sustain safely a certain stress, could resist the same stress if indefinitely repeated. Accordingly, a gradually increasing force was applied to a bar of one square unit cross-section until at a single application the point of rupture for tension, compression, or shear was reached, and the corresponding value t was called the resistance to tension, compression, or shear of the material in question. This value, t, we shall call in future the "carrying strength" (*Tragfestigkeit*) for tension, compression, or shear,† and thus we know that any slowly

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* For the older of these experiments, see Navier, "Résumé des Leçons, etc.," Paris, 1853; as also the very complete work of Morin, "Résistance des Matériaux," Paris, 1853, fourth volume of his "Leçons de Mécanique Pratique." For the more recent, Kirkaldy, "Results of an Experimental Inquiry," etc., London, 1862; also v. Kaven, "Collectaneen," etc., in the Ztschr. d. Hannöv. Arch. u. Ing.-Vereins, 1868, separate imprint by Schmorl u. v. Seefeld. The last-named gives a very full review of the results of experiments.

† The term "breaking strength" (Bruchfestigkeit), used by Launhardt, has thus far been used with too many significations. The word "carrying" implies generally a steady or dead load.

increasing or steady strain, once applied, *less* than t will not rupture the material.

The fact that severe and frequent impact has an especially prejudicial effect has indeed been long admitted, but in the year 1858 it was pointed out by A. Wöhler that, even disregarding the effect of impact, it was still necessary, for a safe foundation for calculation, to experiment upon the resistance of the material to stress frequently repeated.* Soon after, Fairbairn instituted experiments upon a riveted girder.† He applied a load first of $\frac{1}{4}t$, and then of $\frac{1}{3}t$. With $\frac{1}{4}t$ it sustained perfectly well 1,000,000 applications, but upon 313,000 applications of $\frac{1}{3}t$ it broke. From this experiment, however, we can draw no general conclusions, especially with reference to the usual coefficient of safety. The apparatus was so arranged that the girder was subjected to considerable shocks as well as to the action of the load; the influence of each could not be separated, nor was it attempted to make such separation.

During the years 1859 to 1870, Wöhler undertook, at the instance of the Prussian Government, exact and extensive experiments upon iron and steel.‡ The specimens were specially arranged so that all foreign influences, such as in Fairbairn's case, the riveting, were eliminated. The results showed, as was to be expected, that certainly a certain stress t by a single application produced rupture, but also that

* Erbkamms Ztschr. f. Bauwesen, 1858, p. 697.

† Engineer, 1864, or Polyt. Centralblatt, 1865. See also the experiments in Civil Eng. and Arch. Journal, 1860 and 1861, or Ztschr. f. Bauwesen, 1865. These and other experiments have found their way into many German periodicals.

‡ Ztschr. f. Bauwesen, 1860, 1863, 1866, 1870. The contribution of 1870 contains the essential results, and has appeared separately under the title, "Die Festigkeitsversuche mit Eisen und Stahl, Berlin, Ernst u. Korn, 1870."

smaller stresses than t could produce injury if repeated a sufficient number of times. A new point of view was thus definitely attained. The change in the grouping of the molecules, caused by the alternating stress, appeared evidently prejudicial to the resistance of the material. Thus, the injury was the more easily effected the greater the differences in the stress, because, as these increase, the corresponding changes of position of the molecules increase. Thus, Wöhler was able to enunciate a perfectly general law, which may be expressed as follows:

Rupture may be caused, not only by a steady load which exceeds the carrying strength, but also by repeated application of stresses, none of which are equal to this carrying strength. The differences of these stresses are measures of the disturbance of the continuity, in so far as by their increase the minimum stress which is still necessary for rupture diminishes.

By the stress t the material is ruptured by a single application, but a less stress than t can by repetition produce rupture, and the less these stresses the greater the number of repetitions necessary. Inversely, the stress may be greater, the less the number of repetitions contemplated. We see, therefore, that the estimation of the factor of safety of a construction depends greatly upon whether it is to remain in use but a limited time, as in the case of rails, axles, etc., or whether an unlimited life is expected, as in the case of bridges, buildings, etc.

The experiments of Wöhler extend to oft-repeated stresses of tension, bending, and torsion. The resistance to torsion is regarded as resistance to a kind of shear,* in

* See Clebsch, "Theorie der Elasticität fester Körper," 1862, Art. 92; or, Wüllner, "Experimentalphysik," Vol. I., 1874, Arts. 53, 54.

which the shearing forces do not lie in the same plane (Fig. 1). If, then, repetitions of compressive stress do not occur, it may be assumed that the relations in this case are entirely analogous to those in which tensile stress occurs. The case is very different when alternate tension and compression occur. For this state of things, the single case in which the opposite strains are equal has alone been investigated; as to the rest, a void remains which is still unfilled at the present day.

When Wöhler, in the year 1870, left the service of the State, he petitioned the Minister of Trade and Commerce for a continuation of his labors, and at the instance of Reuleaux, Prof. Spangenberg, of the Royal Gewerbeacademie, was intrusted with the task. The experiments of Spangenberg,* which have now lasted about three years, while those of Wöhler extended over twelve, are as yet very limited, but have completely confirmed the law of Wöhler. Spangenberg has directed his attention to other metals, and also especially to the surfaces of fracture produced by different modes of rupture, and endeavored to explain the phenomena of fracture by the aid of his hypothesis as to the molecular constitution of metals. The prosecution of such researches is of importance both theoretically and practically, since up to the present time a general point of view for the estimation of strength indications is almost entirely wanting.

* Spangenberg, "Ueber das Verhalten der Metalle bei wiederholten Anstrengungen," in Erbkamms Ztschr. f. Bauwesen, 1874 and 1875, also separate reprint by Ernst and Korn, 1875.

CHAPTER II.

REMARKS UPON WÖHLER'S LAW.

THE law of Wöhler, in the general form already given, is, without doubt, correct, and it may even be considered as a *long-known* result of experience, since we continually make unconscious use of it. If one endeavors to break a beam walled in at the end with the hand, and a single pull proves insufficient, he naturally ceases, and pulls again and again, and when this fails, perchance, accomplishes the fracture by bending to and fro. The force of the arm is not greater in the second case than in the first, but we do not even need as great a force. We have long known, therefore, that by alternate stress in opposite directions, where the differences of stress are greatest, the force necessary for rupture is less than for repeated stress in a single direction, and still less than for a single application of such a stress. So much the more remarkable then is it that for so long no attention has been paid, even in the most important structures, as to how often and in what manner their component parts are strained. At the same time, it is worthy of notice that the practice already alluded to of Gerber and of the American engineers is based upon a true instinct. Had more attention been devoted to it, the experiments of years might not perhaps have been necessary to give general expression to a law that every one instinctively applies.

There remains still much room for the further develop-

ment of Wöhler's law. In his experiments, the stress was repeated very rapidly; the strains, however, require a certain time in order to reach their full intensity; we disregard now impact proper. What influence now has the rapidity of the repetition, what influence the rapidity of the increase of stress, and what the duration* of the individual stresses? The two last questions are not as yet satisfactorily answered even for the ordinary carrying strength t.

It is not necessary to coincide with the view of Wöhler that the different kinds of strength for iron and steel may be deduced from any one of them. It is sufficient to know that for stresses of a certain kind, and for a given position of the plane of the forces, the law of Wöhler holds good.

We must also keep separate the general expression of the law and those results of experiment by which it has been illustrated by Wöhler. These values, as a matter of course, hold good only for the materials experimented upon. But until now, whenever even a *once* applied load has been considered, no trouble has been taken to ascertain the strength of the material for other conditions of load application; although even for perfectly definite kinds of material, such as rolled iron for bridges and other structures or for iron plate, differences in strength for a once applied steady load, of from thirty to forty and even fifty per cent, are by no means of rare occurrence (Chapter V.). If, a short time since, one had ventured to take exception to this, the answer without doubt would have been, "Have we not for this reason the coefficients of safety?" Well, the coefficients of safety still exist also to-day.

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* Compare Wöhler, "Zeitschr. f. Bauwesen," 1860, p. 240; Spangenberg. Separatausgabe, p. 15.

Although, therefore, without question, many influences still remain to be determined, and the attainment of copious numerical data upon the strength of the various kinds of materials is most pressingly needed, still we possess definitely in Wöhler's law, provisionally in his numerical determinations, the best point of departure for a rational method of calculating the dimensions of the various parts of constructions in iron and steel. The characteristic difference between the new and old methods of calculation, is that the first naturally is not absolutely exact, while the last is without doubt erroneous. (See also Chapter XXIX.)

CHAPTER III.

LAUNHARDT'S FORMULA.*

Let us take now a bar of one square unit cross-section. If we apply a load equal to the carrying strength t, it will produce rupture. Let us assume now the stress to be somewhat less than t; then, according to Wöhler, a certain number of repetitions will be necessary in order to produce rupture. As this stress diminishes, the number of necessary repetitions will increase. Evidently, then, there is a certain stress for which the number of repetitions necessary for rupture will be greater than the number to which the bar in practice can ever be submitted, and which will therefore constitute a perfectly safe stress. This stress, in case the bar, after each application, returns to its primitive unstrained condition, we call the "primitive safe strength," and designate it by u. Then u will be greater the less the number of repetitions to be endured—for instance, greater for rails which are relaid after a certain time than for bridges which are expected to last indefinitely. We shall consider in what follows only the latter case; the general formulæ hold good, however, for all other cases, and then the primitive safe strength will vary between the value given by us and the carrying strength t. According to the definition of the primitive strength, the difference of the strains is $d = u - 0 = u$.

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* Launhardt, "Die Inanspruchnahme des Eisens," Ztschr. d. Hannöv. Arch. u. Ing.-Vereins, 1873, p. 139.

Ordinarily, however, the bar does not return to a perfectly unstrained condition, but there remains a minimum strain c. The stress which in this more general case just causes rupture, we call the "working strength" and designate it by a. The difference of strain is now therefore $d = a - c$, and we have

$$a = c + d \qquad (2)$$

According to Wöhler's law, a decreases as d increases. The limiting values of a are, according to (2) and the definitions of u and t,

$$\begin{array}{ll} \text{for } c = o & a = d = u \\ \text{for } d = o & a = c = t \end{array}$$

Carrying strength and primitive strength are special cases of working strength.

Since a must be a function of d, we can put

$$a = \alpha d \qquad (3)$$

where α is at present an unknown expression. Thus far, however, we know that

$$\begin{array}{llll} \text{for } d = o, & \text{since then } a = t, & \text{by (3) } \alpha = \infty \\ \text{and for } d = u, & \text{since then } a = d, & \text{by (3) } \alpha = 1 \end{array}$$

These conditions satisfy the expression given by Launhardt,

$$\alpha = \frac{t - u}{t - a}$$

which, however, we have still to test for intermediate values by the results of experiment. We have, then, from (2) and (3)

$$a = \frac{t-u}{t-a}\,d = \frac{t-u}{t-a}\,(a-c)$$

or

$$a = u\left(1 + \frac{t-u}{u}\cdot\frac{c}{a}\right) \qquad (4)$$

If, now, the stress upon any member of a construction is B, we have

$$\frac{c}{a} = \frac{min\ B}{max\ B}$$

and, therefore,

$$a = u\left(1 + \frac{t-u}{u}\cdot\frac{min\ B}{max\ B}\right) \qquad I.$$

This is Launhardt's formula. It is applicable when a piece is always extended or always compressed, or generally, submitted to stress of a single kind. The value of u for compression is not yet determined satisfactorily, and meanwhile we take the values of t and u the same both for compression and tension, a practice which seems allowable from certain observations (compare Chapter V.), and which as regards t, has been the custom heretofore.

We must now, therefore, from this point on, form a more general conception of the terms, strength of tension, compression, shear, etc., and understand by them the working strength for the corresponding methods of stress. The *special* values of this working stress, which up to this time have been understood by the above terms, we now designate specially as *carrying strength* for tension, compression, shear, etc.

We have yet to prove whether the expression of Launhardt for the coefficient α holds good for intermediate

values of u and t. For this purpose, we solve (4) for a and obtain

$$a = \frac{u}{2} + \sqrt{\left(\frac{u}{2}\right)^2 + c\left(t - u\right)} \qquad (5)$$

where we must have + before the radical, because a must be positive and greater than u. According to the method of loading and the kind of material, t and u vary, as also a for a given c. Hence, in order that an experiment may possess any value, the results must all be obtained in the same manner and with the same material. The results best suited for comparison are beyond doubt those obtained by Wöhler with Krupp's untempered cast spring-steel,* and it may be said for Launhardt's formula that they agree excellently with them. Thus Wöhler found for the above material under flexure, per square inch (*Rheinisch*),† $t = 1100$ centners, $u = 500$ centners, hence from (5) the working strength is

$$a = 250 + \sqrt{62500 + 600\,c}$$

Below we give the comparison of the formula with the experimental results obtained by Wöhler:

For c	=	0	250	400	600	1100
a by experiment	=	500	700	800	900	1100
a from formula	=	500	711	800	900	1100

According to previous views, the stress of 1100 is that necessary for rupture, but we see from the above that all stresses down to 500 may cause rupture.

* See also the remarks by Spangenberg, "Ueber das Verhalten d. Metalle," p. 11.

† Since we are only concerned with the comparison, the original numbers of Wöhler are retained.

CHAPTER IV.

FORMULA FOR ALTERNATE TENSION AND COMPRESSION.

IT frequently happens that the same piece may be subjected to alternate tension and compression. Since the formula of Launhardt in such case no longer holds good, we shall endeavor here to deduce by a similar process of reasoning and upon the basis of Wöhler's law, a formula applicable to this case also.

Wöhler has investigated the important case in which the opposite stresses are equal. The strength s for this case we call the "vibration safe strength." If the stress in one direction becomes zero, s becomes the primitive safe strength u. Here, then, are two limits indicated.

Let a bar of one square unit cross-section be exposed to alternate compression and tension. Then for every value a for the greater of these two strains there is a corresponding value a' for the less, so that for the greatest number of vibrations which can ever occur between $+a$ and $\mp a'$ the material remains uninjured. The difference of the strains is then $d = a + a'$, or

$$a = d - a' \qquad (6)$$

if only the numerical values are inserted without regard to sign.

Now, according to Wöhler's law, a decreases as d increases, and, in general, a is a function of d.

We can therefore put

$$a = \alpha d \qquad (7)$$

But from (6) and what has been already said,

$$\text{for } a' = o \qquad a = u = d$$
$$\text{for } a' = s \qquad a = s = \tfrac{1}{2} d$$

We have also from (7)

$$\text{for } a = u \qquad \alpha = 1$$
$$\text{for } a = s \qquad \alpha = \tfrac{1}{2}$$

These conditions are fulfilled by the value

$$\alpha = \frac{u - s}{2\,u - s - a}$$

hence we have

$$a = \frac{u - s}{2\,u - s - a}\,d = \frac{u - s}{2\,u - s - a}\,(a + a')$$

or

$$a = u\left(1 - \frac{u - s}{u} \cdot \frac{a'}{a}\right) \qquad (8)$$

If now, for any piece in a structure, *max* B is the greatest occurring stress, whether of tension or compression, and *max* B' the greatest stress in the opposite direction, we have

$$\frac{a'}{a} = \frac{max\ B'}{max\ B}$$

and accordingly,

$$a = u\left(1 - \frac{u - s}{u} \cdot \frac{max\ B'}{max\ B}\right) \qquad II.$$

Here also we may call a the working strength. All quantities are simply to be inserted numerically without reference

to their signs before insertion. The primitive strength is always u, and a the working strength in the direction of the absolute greatest stress *max B*. Since u is not yet known for compression, we may, as in Launhardt's formula, for the present, use its value for tension, which is rather too small if any thing.

In many constructions, the vibrations occur between the limiting strains a, a' for primitive stress of zero. In others we have a previous stress c, in most cases due to dead weight or weight of the structure itself. However, then, we conceive it to be at the beginning, the action of each complete vibration must be similar, neither can it be essentially changed by the long-continued action of c, which lies far within the elastic limits.

The formulæ I. and II. hold good not only for tensile and compressive stresses, but also for all other kinds; the proper values for t, u, and s being used for each case.

If, generally, we denote by ϕ the ratio of the two limiting stresses of a piece, the less to the greater, without reference to sign, our formulæ become

$$a = u\left(1 + \frac{t - u}{u}\phi\right) \qquad (Ia)$$

for stress in one direction only, and

$$a = u\left(1 - \frac{u - s}{u}\phi\right) \qquad (IIa)$$

for alternate stresses in opposite directions.

CHAPTER V.

CARRYING STRENGTH FOR COMPRESSION AND TENSION.*

THE older experiments upon wrought iron show a greater uniformity and give, upon the whole, greater values for the carrying strength than the new. Navier quotes the results of seven experimenters in France, England, and Italy, who give as mean values per square centimetre, 3940, 4220, 4290, 4450, 4610, 4680, 5010 kil. (700 kil. per sq. cent. = very nearly 10,000 lbs. per sq. inch).

The carrying strength depends, other things being the same, upon the method of manufacture. For round and square iron, Kirkaldy found as a mean of very numerous experiments 4050 (varying from 3780 to 4330); Wöhler found for round iron from the Borsig and Königs Hütten, 4110 (from 3730 to 4530); Knut Styffe found for soft puddled iron a mean value of 3400 for round iron and 3460 for square iron; all in kilogrammes per sq. centimetre.

Styffe gives for seventeen tests of English rolled iron from three different establishments, a mean of 3910 (2940 to 5100); for sixteen tests of Swedish rolled iron from four establishments, 3760 (3170 to 4900). Bauschinger found for rolled iron from Wasseralfingen, 3890 (3750 to 4140); for angle iron from the Lothringen Works, 3195. The mean of many experiments upon angle iron by Kirkaldy was 3850 (2910 to 4310).

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* Arts. 5 to 12 are not necessary for the problem of dimension determination, and may, if found to be destitute of interest, be omitted.

For rivet iron from Borsig, Wöhler found from two tests 5120; for English homogeneous iron from three tests, 4280. Styffe gives for a strip from the head of an English rail, as a mean of three tests, 3380; for another strip from the stem, two tests, 3090; for another strip from a wheel tire of Low Moor iron, 3760. Bauschinger found for gas-pipe, transverse to the direction of rolling, 1400 to 1500.

Styffe gives the carrying strength for tension of soft iron as 3380; Gerber and many others assume in bridge construction, 3500; Reuleaux gives 4000; Von Kaven deduces from Kirkaldy's experiments the mean value of 4200. For good iron, such as should always be used in bridge construction, the tensile carrying strength lies generally between 3500 and 4000 (see also Chapter XII.). Figured or ornamental irons, difficult to roll, give in general smaller values and show inferior tenacity. Their use should therefore so far as possible be avoided.*

Navier gives for iron wire, such as used in suspension-cables, according to the experiments of Buffon, Telford, and Seguin, the mean values 6000, 6360, 6000, respectively. Moseley considers 6580 as allowable, Reuleaux 7000, Von Kaven deduces from Kirkaldy's results the mean value 6700, Laissle and Schübler take for decreasing diameter from 5000 to 8000.† As a mean, we may take 6000, but should always have recourse to direct experiments.

* See the Memoir of Keck in the "Zeitschr. d. Hannöv. Arch.- u. Ing.-Vereins," 1867, p. 395, issued separately by Schmorl and Seefeld, under the title "Ueber das zur Brückenconstr. zu verwendende Schmiedeeisen, Blech und Façoneisen."

† Karmarsch has expressed the relation between diameter and strength by a formula. "Ueber d. abs. Fest. von Metalldrähten," Dingler's Poly. Jour., 1859, Vol. 154; "Ueber die Festigkeit von Drahtseilen," Polyt. Centralblatt, 1853, 1862, 1863, 1865.

The tensile strength of iron plate is in general less than for other kinds of iron, and there is often a remarkable difference according as the stress is applied in the direction of the fibre or transverse to it. The value in the first case is generally greater than in the second. The same holds good for the kinds of iron used in bridge construction, but since the stress here is generally in the direction of the fibre, it possesses less interest.

Kirkaldy found as a mean of many experiments, 3570 (3210 to 3870) with the fibre, and 3250 (2920 to 3550) across. Fairbairn, on the other hand, found for four kinds of boiler-plate 3540 (3080 to 4060) with the fibre, and 3620 (2940 to 4330) across. Bauschinger found from different boiler-plates from the Steinhauser Hütte as a mean of twelve experiments, with the fibre 2820 (2600 to 3270), and across 2730 (2350 to 3180).

Boiler-plate from the locomotive "Fugger," which exploded at Würzburg, gave for the uninjured portions, 3040 with and 2880 across the fibre. Stevens found for the best English Low Moor boiler-plate as a mean of five experiments, with the fibre 4140 (3890 to 4500); for a plate designed for water-tanks, from six experiments, 2900 (2320 to 3670).

In a specimen of very distinct fibre, Bauschinger observed 2910 with and 1910 across. The difference in question appears to depend, however, upon the method of production as well as the fibrous character and rolling. Experiments in the establishment of Gouin & Co., in Paris, also gave a greater carrying strength with than across the fibre, but only $\frac{1}{16}$th greater for charcoal iron and $\frac{1}{8}$ greater for coke iron.

Von Kaven deduces from Kirkaldy's experiments, for iron plate, the mean value 3800. The English Admiralty

requires for first quality 3460 with and 2830 across the fibre; for second quality, 3150 with, 2680 across. Without preferring special experiments, we cannot in general assume over 3000 with and 2700 across fibre. The ratio $\frac{9}{10}$ of cross to with, agrees well with the mean of Kirkaldy and with the experiments of Edwin Clark.

For steel, the tensile carrying strength depends especially upon the amount of carbon and other constituents, which will receive notice hereafter. Since the proportion of carbon is not always known, we give here only a few general data.

Kirkaldy found for various steels, from nine different works, a mean value of 6770, from 4930 for a kind of puddle steel, to 9340 for cast steel; Sheffield Bessemer gave 7840. Wöhler found for cast-steel axles from Krupp, Borsig, Vickers, and Bochum, as a mean of twelve experiments, 6250 (from 4020 for Vickers up to 7670 for Krupp); also for heads of Krupp's cast-steel rails 7380, and for tool steel from Firth, 8400. Styffe found for hammered Bessemer round steel, containing from 0·86 to 1·35 per cent carbon, as a mean of eight tests, 7730 (from 6880 to 8970); also for rolled Bessemer steel, square and round, containing from 0·38 to 1·39 per cent carbon, by nine tests, 6480 (from 4550 to 9840), and for rolled Swedish cast steel, round, 0·69 to 1·22 per cent carbon, by four tests, 8910 (from 7280 to 10170). We may assume for puddle steel about 5000; for good medium Bessemer steel, 5500 to 6500; for very good and hard cast steel, 8000. The last value is given also by Reuleaux, Laissle, Schübler, and others. For styrian cast-steel plate (Bessemer), Bauschinger found as a mean of two experiments, 5025 with and 5180 across the fibre. Wöhler ob-

tained for cast-steel plate from Krupp, by five experiments, 5390 with (4900 to 5770), and for the same from Borsig from one experiment, 5040. Tresca observed for two kinds of cast-steel plate, 5400 with and 5760 against. Stevens, from six experiments with the best English Bessemer steel, obtained 5880 (from 5240 to 6090). We may therefore assume safely for steel plate both with and across the fibre, 5000.

There exist hardly any data upon the compressive carrying strength, and a practically applicable definition of it is hard to give. Bauschinger found by experiments with steel that rupture of the material by compression alone was hardly possible, and the statement of Rondelet appeared confirmed, that the material fails rather by flexure than crushing, when the height is more than three times the least dimension of cross-section (Chapter IX.). Rondelet, and after him Navier, gave the compressive carrying strength for wrought-iron as 4950. Moseley gives 6580. According to Bauschinger's experiments with Bessemer steel, the crushing strength is considerably greater than the tensile. Since also, from the oft-repeated experiments of Wöhler and Spangenberg upon flexure, it appears that rupture occurs always upon the tensile side, it is certain that the material does not fail by compression sooner than by tension, and we can safely assume therefore the working strength the same both for tension and compression. It is, however, assumed that buckling of the compressed parts does not occur. Fairbairn observed, in a number of experiments with plate beams, the beginning of rupture always in the upper flange; it is now the custom to give careful attention to the stiffening of the pieces, and where it is requisite, to stiffen the flange.

CHAPTER VI.

TRANSGRESSION OF THE ELASTIC LIMIT.

THE limit of elasticity is ordinarily defined as that stress per square unit just beyond which permanent change of form occurs, while for smaller stresses the body returns, upon removal of the stress, completely to its primitive condition. The stress is always to be considered as gradually increasing and not suddenly applied, liable to sudden variations or of an impulsive character. The definition is, however, destitute of *theoretical* value, as so definite and sharply defined a limit is neither probable nor indicated by experiment. On the contrary, Hodgkinson and Clark have observed permanent set even for very small stresses.* Provisionally, we may content ourselves by defining the elastic limit with Fairbairn, as that stress below which the change of form is, approximately, directly proportional to the force, while above it the change of form increases much more rapidly than the force. The expressions "approximately" and "much" are by no means so indefinite here as may be supposed. The experiments of Bauschinger upon tension, compression, flexure, and torsion,† in every case indicated

* Förster's "Bauzeit," 1853, p. 197; "Ztschr. d. Hannöv. Arch. u. Ing.-Vereins," 1865, p. 437.

† Bauschinger, "Mittheil. a. d. mech.-techn. Laborat. der k. polyt. Schule zu München," Ztschr. d. bair. Arch. u. Ing.-Vereins, 1873. The contributions of Bauschinger, which are not confined to metals only, were regularly published in the last journal as well as separately issued in Munich by Oldenbourg.

very precisely the elastic limit; for example, for tension, "where, for the same increment of the load, all at once a disproportionate extension occurred, the maximum of which was only attained after some time." This sudden extension is to be attributed almost entirely to permanent changes of form; the transitory or *non*-permanent changes remain proportional to the stress until very near the limit of rupture, and the coefficient of elasticity is found to be always almost entirely independent of the stress (Chapter IX.). By the first definition above, of the elastic limit, the permanent changes of form occurring in Bauschinger's case are neglected.

All experiments thus far made indicate that when the limit of elasticity is exceeded, the tensile carrying strength is considerably increased, but the ductility and tenacity are diminished, the material becomes brittle and incapable of resisting shocks. According to experiments made in the Arsenal at Woolwich, an iron rod four times broken gave for t the successive values, 3520, 3803, 3978, 4186. Bauschinger broke a piece of iron seven times and found an increase of the carrying strength of from 3200 to 4400. Paget found for iron chains, that, after distention, they sustained a greater dead load, but offered less resistance to shocks.* Fairbairn endeavored to account for these facts by the theory that at first all the particles were not brought into play, but, as in ropes, came into play little by little under sufficient loading. The observation of Bauschinger, that the increase of carrying strength was especially evident and regular for rolled iron and for stress in the direction of

* "The Engineer," 1864, p. 287.

the fibres, goes to confirm this view, and sustains the comparison with a rope. The analogy holds good also in regard to the diminished resistance to shock, as a strained rope comports itself similarly in this respect. It also explains why a bar breaks more readily by a suddenly applied stress (without impact) than by a gradually increasing one; in the first case fewer particles are brought into play.

Further, by exceeding the elastic limit, that limit is itself increased. Tresca has been able, in his experiments upon flexure with iron rails, to extend the limit of elasticity nearly up to the point of rupture, during which the coefficient of elasticity diminished only $\frac{1}{10}$th.* It has long been the accepted practice to take as the allowable stress, *b*, a certain fraction of the elastic limit. In such case, as in the case of the carrying strength, *b* will be greater the oftener the material is severely strained. But, at the same time, it becomes more brittle, therefore less capable of resisting shocks and local excesses of the elastic limits, such as in many constructions, especially in bridges, are not infrequent. We do not need to conclude from the foregoing, as has recently been done in many quarters, that a test of the metal beyond the elastic limit is advantageous. It is also well to observe that the increase of carrying strength by each transgression of the elastic limits cannot go on indefinitely; a decrease must again occur, unless we are ready to admit that for very gradual increments of the individual stresses and greater intervals between their application, the primitive strength is greater than the original carrying strength.

If, now, the transgression of the elastic limits can act injuriously, it ought not, at least designedly, to occur. In

* "Comptes Rendus," vol. 173, p. 1153.

this connection, however, it is sufficient to know, from the numerous experiments of Styffe* and others with the most different varieties of iron and steel, that the ratio of the elastic limit to carrying strength lies ordinarily between $\frac{1}{1.4}$ and $\frac{1}{1.8}$, and even under the most unfavorable circumstances rarely reaches $\frac{1}{2}$, and that we always assume considerably smaller stresses as allowable.

Wertheim and Styffe have sought to obtain a more precise definition for the elastic limit.† As it is, however, not more scientific in its basis, nor more useful in practice, it would be superfluous here to discuss it. Since the experiments of Hodgkinson and Clark, we can regard the elastic limit as only significant from a practical point of view, and even in this sense as very limited in its application. The elastic limit as defined, leaves us entirely in the lurch, when we have to consider rapid changes of strain and oft-repeated stresses.

As to the influence of the duration of a dead load, Vicat had already thirty years ago instituted experiments. He loaded wires during thirty-three months up to three fourths of the carrying strength, whereupon the heaviest loaded broke. Vicat concluded from this, and from the fact that the elongations appeared to increase proportionally with the time, that any load beyond the elastic limit would in a sufficiently long time produce rupture. If we consider that even very small stresses cause permanent changes of form, it would appear more correct to conclude that *every* and *any* load will in time cause rupture. This, Fairbairn‡ considered to be indicated

* Knut Styffe, "Die Festigkeitseigenschaften von Eisen und Stahl," from Sandberg's English translation, by C. M. v. Weber, Weimar, 1870.

† Poggendorff's Ann., Ergänzungsband II., Styffe, Chap. I., Art. 12.

‡ Fairbairn, "Die eisernen Träger," German by Brauns, Braunschweig, 1859, p. 26.

by his experiments upon cast-iron girders. An examination, however, of his results by no means justifies this conclusion. On the other hand, we may conclude, from the circumstance that the carrying strength increases as the stress exceeds the elastic limit, that the safety for a dead load increases with its duration. Here again it may be objected that a decrease of the carrying strength may succeed the increase, and thus, all things considered, we can only admit that the influence of the duration of a dead load is as yet unknown. In such case, no notice can be taken of it.

That every load, on the other hand, requires a certain time in order to cause its corresponding change of form, has been recognized ever since Hodgkinson and Wertheim; it agrees also with Fairbairn's analogy with the rope, and has been again confirmed by Bauschinger. The same holds true for the transitory changes of form, and, moreover, a rod subjected only to a transitory change of form does not, upon removal of the load, at once *perfectly* recover its former condition; there occurs a so-called "secondary action." This was observed by Kupffer even after several days.*

Thurston considers it as a new phenomenon,† that the carrying strength and elastic limits after twenty-four hours' steady strain are thereby increased. There appears, however, in this nothing new. That the carrying strength of iron and steel is increased by the passage of an electric current, and that the ductility may be affected now in one and now in the other direction by immersion in acid, would appear to be indicated by solitary experiments, which, however, lack indeed confirmation.

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* Kupffer, "Recherches exp. sur l'élast. des Métaux," St. Petersbourg, 1860.

† See APPENDIX: Thurston on the "Exaltation of the Elastic Limit by Strain."

CHAPTER VII.

MECHANICAL TREATMENT—ANNEALING, TEMPERING

By exceeding the elastic limit, that limit is itself increased as well as the carrying strength; the ductility and tenacity decrease. Since now by rolling, hammering, stretching, the limits of elasticity at the portions operated upon are exceeded—there are, indeed, very considerable changes of form—the necessary effect of such mechanical treatment is already clear.

Annealing, or heating and slowly cooling, has an effect precisely opposite; the material becomes more ductile, and its carrying strength diminishes. According to Tunner, the brittleness of material hardened by mechanical treatment gradually disappears by simple quiescence.* Thus a wire, which immediately after leaving the draw-plate broke when bent at an obtuse angle, regained workability in the course of a few days, and still more after a few weeks.

Kirkaldy's experiments have shown quite clearly that cold rolling increases the carrying strength very considerably. Thus t was increased by this means nearly double, from 3220 to 6260; by annealing it was reduced to 3580. Styffe caused a previously annealed and thereby softened iron rod to be cold hammered until its cross-section was reduced by one half; the carrying strength was thereby increased from 3140

* "Wochenschr. d. niederöstr. Gewerbevereins," 1874, p. 245; "Hannöv. Ztschr.," 1847, p. 618.

to 5830. According to Kick, the cold-rolled iron in use in the United States is much more brittle than ordinary. It has been observed that the carrying strength of cold-rolled material is diminished by removing the outer surface or skin. These and many other apparently independent facts are all explained in accordance with the view already set forth.

If the mechanical treatment takes place when heated, two opposite influences are called simultaneously into action—transgression of the elastic limit and annealing; these may partly or wholly neutralize each other, and the material may, with undiminished or even increased ductility, gain in strength. In England, we find accordingly very frequently, repeated reworking.* If the opposing treatments follow each other, the opposite effects will seldom just neutralize. According to Kirkaldy, the carrying strength, after rolling and then annealing, was always increased; the influence of the rolling was greatest.

A body once annealed will only be again affected by a higher temperature, provided, of course, that in the mean time it has undergone no treatment having an opposite effect. It follows, therefore, that the effect of annealing will be so much the greater the higher the temperature in comparison to that at which the previous mechanical treatment took place. This agrees also with the observations of Styffe.

Tempering has the same effect as transgression of the elastic limits, for steel as well as iron,† only for steel, the carrying strength, elastic limits, and also the brittleness are

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* V. Kaven, "Collectaneen,' Cap. XII.

† Styffe, in another place, p. 53.

much more increased. For many purposes, hardened material possessing little resistance to shocks is inapplicable. Tempering consists in suddenly cooling the glowing material by plunging it into a suitable bath, ordinarily oil or water, according as the material is steel or iron. The brittleness can be somewhat reduced by moderate heating (" drawing"); by annealing, it, together with all other effects of tempering, can be again removed. The effect of tempering is much greater for steel than iron, but depends in both upon the chemical composition and many other circumstances.

Tresca increased* the carrying strength of two kinds of steel plate, by tempering, from 5400 to 8784 and from 5764 to 8880 respectively. Wöhler cut bars from a hardened cast-steel axle, and found for one 9209 carrying strength, and for the other previously annealed, 7455. Numerous experimental results upon all the effects of tempering are given by Kirkaldy, with which those given by Styffe (Table VI.) agree in the main, so far as they go. It has been shown by Wöhler, Hensinger v. Waldegg, and others, that a slight contraction occurs in tempering. The contraction of a steel bar of 33 mm. (1.3 in.) thickness, was, according to Wöhler, 1 mm. per metre in length (0.012 in. per foot).

Upon the strength of weldings, we have experiments by Kirkaldy. The diminution of the tensile carrying strength is placed between 2.6 and 43.8 per cent, and the ductility was also considerably diminished, especially for steel. According to Nasmyth, the strength of the weld depends essentially upon whether the necessary flux is completely driven out

* "Ann. des mines," 1861.

again. In bridge construction, welds in the principal members are not allowable.

In screw-cutting, also, there is a diminution of the carrying strength, amounting, according to Kirkaldy, to 30 per cent. We may find an explanation in the fact, that by the cutting the hardened surface is removed; it may be also because of small crevices caused by sharp dies. The last circumstance, as well as the hardening due to the greater expenditure of force, may explain the fact that Kirkaldy found a greater strength for screws cut with blunt dies than for those cut with sharp. If, in general, the strength for screw-bolts of small diameter is found somewhat greater, it need excite no astonishment, since Kirkaldy has found that universally the strength increases with diminishing diameter, as, indeed, by reason of the relatively greater influence of the rolling, might have been expected.

CHAPTER VIII.

INFLUENCE OF FORM.

THE *form* of a construction-piece may exert a very marked influence upon its strength. The bar shown in Fig. 2 will carry less per square unit of cross-section than when bounded by the dotted line. For the load to the right of this line can only be transmitted by the fibres to those on the left, and these last must therefore sustain more than the mean stress per square unit, and rupture will occur sooner than otherwise would be the case. By the flexure which in this case is also produced, the stress will be still more increased, and the arrangement is unfavorable even when the load acts upon the smaller end. (Compare Chap. XVIII.)

The above explains why Wöhler found the strength for bars with sharp offsets noticeably less than for those with curved; in the latter case (for a *cavetto*) the transference of the load is gradual. The strength in the first case was often $\frac{3}{5}$ to $\frac{4}{5}$ as great as in the second under similar circumstances, but the experiments do not at present suffice for definite data, as evidently the degree of change of the cross-section and all the influences indicated in the preceding paragraph also come into play. That rail chairs have much more influence upon the resistance than ordinarily supposed, has been shown by the experiments of Tietze.* Turned axles also, which are subjected principally to a strain of torsion,

* See "Die Ergebnisse der Schlagprobe," Tech. Blätter, 1874, p. 20.

show an analogous loss of strength.* We should then, wherever possible, avoid sharp angles, and replace them by a gradual curve.

It is remarkable that a bar, as in Fig. 3, will sustain a greater dead load than when the entire bar has the smaller diameter.† According to what we have said, we should expect the opposite. A very short steel rod grooved as above, bore, according to Vickers' experiments, 12,500 kilog. per square centimetre, while one 35 cm. long (13.8 inches) of the same material, of same diameter throughout, bore only 9440. From the experiments of Kirkaldy, very short grooves of only about $\frac{2}{3}$ the diameter in height caused an increase of t for tension of about $\frac{1}{8}$.

At Woolwich, the carrying strength of Bessemer steel was always found much greater than by Kirkaldy; the specimens were grooved transversely. These facts are hard to explain, but may perhaps be thus accounted for: A bar submitted to tension bends under great load by reason of the want of homogeneity of the material, which is sometimes quite apparent.‡ The bending stress contributes to the rupture, but is less the shorter the grooved space. If this explanation is accepted, it follows that in general a very short rod should carry a greater dead load than a longer one of the same material and cross-section. Whether this is the case, we know not. Further, the stress for compression should show a similar difference. This, according to Bauschinger and others (Chap. IX.), is found to be the case.

* Köpke, "Ueber d. Fest. eingedrehter Axen, Hannöv. Ztschr.," 1864, p. 220.

† Compare v. Kaven, "Collectaneen," Cap. IV.

‡ This has been especially observed and noticed by Styffe, p. 18.

Almost all the experiments thus far instituted, even those of Wöhler, have been made with finished bars. Fairbairn alone has investigated riveted girders, for the special purpose of comparing different cross-sections.* The girder failed almost always by side buckling or by crushing of the upper flange, an effect which is now prevented by a stiff form of flanges, and by means of additional stiffening material in the shape of vertical angle or T-iron placed at proper intervals. How, then, the strength of the pieces in a structure compares with the single piece, is nowhere determined. It is, however, certain that this depends very essentially upon the character of the connections, and hence careful attention should be paid to their arrangement and execution.

* Fairbairn, "Die eisernen Träger," by Breuner, 1859.

CHAPTER IX.

VARIOUS CONSTITUENTS—AMOUNT OF CARBON.

THE ideas, wrought-iron, steel, and cast-iron, are just at present rather indefinite, and more easily *felt* than expressed; a definition which should be correct to-day would probably be found defective to-morrow. In very recent works, we still find that wrought-iron should contain up to $\frac{2}{3}$, steel from $\frac{2}{3}$ to 2, and cast-iron over 2 per cent of carbon. But we have now steel with $\frac{1}{4}$ per cent and less carbon, and wrought-iron even as high as 1 per cent. It is elsewhere laid down, that steel can be tempered and wrought-iron not, but steel rich in phosphorus and poor in carbon cannot be tempered, while wrought and even cast iron may be under certain circumstances.*

Greiner, the director of the Bessemer Works at Seraing, and Philippart, give, in the *Ingenieurverein* at Lüttich, the following definition of steel in opposition to wrought-iron:* "Under the head of steel, we may include every kind of iron produced from a fluid form, and which, by reason of the homogeneous character and compactness thus obtained, has a greater resisting power; and which also, from the method

* See Benedict, "Ueber d. Definition v. Schmiedeeisen und Stahl," "Ztschr. d. östr. Ing.- u. Arch.-Vereins." 1875, p. 345.

of production, is more uniform in its composition and deportment." According to this, the name of steel must be refused to many products classed under that head.

For cast-iron, the definition given by Benedict may, in the main, still hold: "By cast-iron, we mean every kind of iron produced directly from the ore, which will not work or weld, which melts at a relatively low temperature, and has the greatest impurities and amount of carbon." No single point of view is sufficient in itself, as, for instance, by the Siemens process, so well known since the Vienna Exposition, wrought-iron and steel may be produced directly from the ore.

Chemically pure iron has thus far been obtained only in small quantities; it is found to be very soft or very brittle, and is melted with difficulty. Carbon gives to the iron the qualities which make it practically useful. It occurs in irons used in structures, from 0·1 to 6 per cent; sometimes in chemical union and sometimes as graphite. If now we consider those groups which go by the name of wrought-iron and steel, we may say that for each, the increase of carbon has a similar effect upon the strength as mechanical treatment or transgression of the elastic limits; the hardness and strength increase, the ductility and resistance to impact and rapid stresses beyond the elastic limits decrease. This is less noticeable in wrought-iron, because marked by the influence of other constituents and by the mechanical treatment. But there is also for steel a limit, beyond which the strength, at least compressive and tensile, again decreases, but at the same time the ductility likewise, so that the characteristics of the material approach more nearly to cast-

iron. The position of this limit depends likewise upon the presence of other elements, as well as upon the influences considered in Arts. 6 and 7. Knut Styffe considers that he has found the maximum tensile strength for iron and puddle steel to occur for 0·8 per cent;* for Bessemer and Uchatius steel, 1·2 per cent. This last agrees with the experiments of Vickers, at Sheffield, which give the maximum for 1·25 per cent.† According to Karsten,‡ steel of 1·0 to 1·5 per cent carbon tempers best, and possesses the greatest tensile carrying strength. For a greater percentage of carbon, it becomes harder, but the strength diminishes; for 1·75 per cent it scarcely welds; for 1·8 per cent it can be worked cold under the hammer only with the greatest difficulty; for 1·9 per cent it cannot be worked, and for about 2 per cent the characteristics attain the limit between steel and cast-iron; the material cannot be drawn out at a white heat without cracking, and falls to pieces under the hammer. These values of course have little pretension to accuracy, and the material is assumed uninfluenced in its properties by foreign substances.

Bauschinger has made very admirable experiments with the Ternitz Bessemer steel.§ The specimens were specially prepared, of the same sort, but containing different

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* Styffe, p. 49.

† Vickers, "Résistance de l'Acier, rélat. aux diff. prop. de Carbone qu'il cont., Bull. de la Soc. d'Encourag.," 1863, Vol. X., p. 561; "Ztschr. d. V. deutsch. Ing.," 1863, p. 105.

‡ Percy, "Metallurgy." Ger. by Knapp and Wedding. Vol. II., 1865, p. 145.

§ Bauschinger, "Versuche ueber d. Fest. des Bessemerstahls v. verschied. Kohlenstoffgehalt," Ztschr. d. bair. Arch. u. Ing.-Vereins, 1873, p. 81.

amounts of spiegeleisen. He found the following carrying strengths:

Carbon per Cent.	Tension.	Formula. (9)	Compression.	Shear.	Flexure.
0·14	4430	4435	4780	3410	7920*
0·19	4785	4510	5390	3710	8600*
0·46	5330	5270	6330	3585	8340
0·51	5600	5480	7000	4020	9300
0·54	5560	5620	6110	3930	8550
0·55	5650	5665	6170	4000	8825
0·57	5605	5765	6550	3645	9600
0·66	6295	6245	6550	4280	8600
0·78	6470	6995	7305	4140	8750
0·80	7230	7134	9670	4820	7645
0·87	7335	7640	8940	5000	7650
0·96	8305	8340	9890	5820	8480

The elastic limits increased in a similar manner from 2950 to 4870, 2775 to 5000, 3750 to 4725.

If we take the tensile carrying strength as ordinate for the corresponding carbon percentage as abscissa, we have a number of points, as given in Fig. 4, which are well embraced by the curved line, which corresponds closely to the equation

$$t = 4350\,(1 + K^2) \qquad (9)$$

where K is the carbon percentage. By means of this formula, the values in the third column are computed. In Fig. 4 we

* These two experiments remain incomplete, and the fibre-strains at the moment of rupture are given.

have also represented in similar manner the other experimental results thus far obtained. The notation is as follows:

+, the results of Vickers.

O, those of Styffe, with hammered Swedish Bessemer round steel, from Hogbö.

□, those of Styffe, with rolled Swedish Bessemer square steel, from Carlsdal.

O*u*, those of Styffe, with rolled Swedish Uchatius cast-steel round, from Wykmannshyttan.

O*k*, those of Styffe, with soft hammered Krupp cast axle-steel.

□*t*, those of Bauschinger, with rectangular tension bars of Ternitz Bessemer steel.

O*t*, those of Bauschinger, with round bars of Ternitz Bessemer steel.

We see from the figure that formula (9) not only gives very closely the results of Bauschinger, but, in general, the mean tensile carrying strength with tolerable accuracy. We see also, however, that various influences can cause by no means insignificant variations. The equation

$$t = 3700(1 + k) \qquad (10)$$

which corresponds to the curve II., gives now a value, below which, ordinarily, the carrying strength will not fall.*

* The results of Vickers may be embraced by the following formula (line III.):

$$t = 2600 + 6700K$$

Thurston assumes

$$t = 4200 + 4900K$$

which gives certainly too large values (line IV.). Haswell has deduced from

As to the results for compression contained in the preceding table, we may remark that the specimens of 3 by 3 by 9 cm. were compressed between two plates. By increasing load an S-shaped flexure was observed, which increased more and more until the prism sprang out. The fibre strain at the moment of springing out is taken as the carrying strength. For the end in view—namely, investigation of the influence of varying proportions of carbon—this is allowable; somewhat longer prisms of the same cross-section would, however, have sprung out sooner.

Bauschinger also tested specimens of all the above steels, of the shape shown in Fig. 5, for which, therefore, the inner prism was relatively shorter, and held perfectly firm. As the load increased, a pressure was reached for which, without further increase of the load, the prism continually diminished in length (up to less than half the height), while the cross-section increased. The stress at this point per square unit of cross-section, which Bauschinger took as the compressive strength, increased with the carbon percentage in a manner entirely similar to the tensile strength as noted above, but from 9250 to 17800, or the latter is always double the first. On the other hand, the elastic limits were found to be independent of the method of experiment. We may, indeed, in general, load very short steel prisms safely up to double the allowable stress for tension; but, as already indicated, for alternate tension and compression, must take

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Styffe's experiments, for hammered and rolled Swedish Bessemer steel, respectively:

$$t = 3500 + 4500K, \qquad t = 3000 + 5000K$$

The choice of the first expression is hardly warranted by the above-named experiments. See APPENDIX.

the allowable stress under similar circumstances, the same for each.

Similarly to carbon, phosphorus also raises the elastic limits and tensile strength, but diminishes the resistance to impact and variation of stress. The iron becomes brittle, coarse-grained crystalline, and breaks easily when cold worked. It is called "cold short." For these and other reasons (Chapter IX.), it is unsuitable for bridge constructions. The effect of phosphorus upon steel is still more prejudicial than upon iron. According to Greiner,* a steel containing from 0·2 to 0·25 per cent phosphorus is unfit generally for technical uses. Steel containing phosphorus is, according to him, best suited for rail-heads, because it diminishes the wear, but it is necessary to diminish the carbon in order that both may not combine to increase the brittleness.

Silica, according to Sandberg in Sweden, and Tunner in Austria, exerts an influence similar to carbon, while Haswell considers it, in the case of steel containing a certain amount of phosphorus, as partially neutralizing the bad effect of the latter. Slag exerts upon phosphoric iron a favorable influence, so far as it diminishes the brittleness, but the iron becomes harder to work without splitting and crumbling. Sulphur is, next to phosphorus, the most injurious ingredient, and acts similarly to the latter, except that it causes the material to work with difficulty when hot, or makes it "red short." Manganese also exerts a prejudicial influence.

The effects of the above and other constituents upon the strength of iron and steel are not yet clearly understood;

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* Greiner, "Ueber phosphorhalt. Stahl," Dingler's Polyt. Journ., 1875, Vol. 217, p. 33; from *Revue Univers*, Vol. XXXV. p. 613.

their metallurgical significance will be found discussed in every text-book upon metallurgy, as well as in the already quoted work by Percy, or in that by Wedding, "Die Darstellung des schmiedbaren Eisens, Braunschweig," now in press.

Whether, in any case, steel or iron is to be preferred, depends upon the relative importance which we set upon strength under dead load, resistance to impact, deportment under different temperatures, cheapness, etc. Generally, there is little to be said, and to discuss special cases would carry us too far.* In the use of steel, the proper percentage of carbon is dependent not only upon the destined use and necessary mechanical treatment, but also upon the composition of the ore and the method of production,† because upon these the other constituents depend. Thus Vickers found, for bodies subjected to both tension and impact, 0·62 to 0·75 per cent; Styffe, for axles of Swedish steel, according as they are welded or in a single piece, 0·4 to 0·6; Greiner, for Bessemer steel axles from Seraing, 0·3; from Krupp, for axles of locomotives and steam-vessels, 0·5 to 0·6; for passenger-car axles, somewhat over 0·6. Greiner gives further for Bessemer steel from Seraing, for boilers, cranks, connecting-rods, 0·25 to 0·35; for tires without weld, and piston-rods, 0·35 to 0·45; for steel rails, 0·4; for springs, 0·45 per cent carbon.

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* See Styffe, Chap. II.: Relative Value of Steel and Iron; further, in App. v. Sandberg: Steel *versus* Iron; also, v. Kaven, "Collectaneen," Chap. IX.

† Compare Robert S. Haswell, "Studien über Bessemer Steel," Technische Blätter, 1874, p. 111.

CHAPTER X.

INFLUENCE OF TEMPERATURE.

THE influence of different temperatures upon the strength of iron and steel, is not yet satisfactorily understood. Only as regards the carrying strength, and by means of numberless experiments, has unanimity of opinion been attained. Accordingly, the carrying strength for most kinds of material appears to increase somewhat as the temperature sinks below 0° C., but there is also a maximum, especially for iron, above 100° C. Within a certain range, from the ordinary temperature of 16° C. (about 60° F.), the carrying strength is almost constant; the beginning and rapidity of increase, as well as the maximum, are dependent upon the influences already considered.

Fairbairn observed in tensile experiments upon iron bars, the carrying strength at 0° in one case the same, in another case one per cent greater than for + 60°. Thurston, in experiments upon torsion, was able to distinguish a perceptible increase of the carrying strength down to — 12°. Spence found by experiments upon flexure with cast-iron even for — 18° C. (about 0° F.), a carrying strength of 3·5 per cent greater than for 15° C. (60° F.). For higher temperatures, Fairbairn's experiments* with iron bolts give the maximum tensile strength at about 163° C. (325° F.), 41 per

* "Ztschr. d. Vereins dtsch. Ing.," 1859, p. 265.

cent greater than for 18° C. (64° F.); later experiments with bar-iron gave the maximum at 213° (415° F.). A commission of the Franklin Institute of Philadelphia found the maximum strength 15 per cent greater than the ordinary; the temperature corresponding was not exactly determined, but was certainly below 288° (550° F.). Styffe has instituted many experiments (see his Table VIII.).

Above the maximum, the strength decreases at first slowly, and with still increasing temperature more rapidly. In this respect also, the various kinds of materials comport themselves differently, and the decrease in general may perhaps be said to take place earlier and more rapidly, the lower the temperature at which the material was subjected to mechanical treatment. The tensile carrying strength decreased, according to Fairbairn, from 202° C. (395° F.), down to a, in the dark, just visible red heat, by 34 per cent. Experiments made by the Commission of the Franklin Institute, just quoted, show a decrease of the tensile strength to 0·66 for 575°, and 0·33 for 700° C., and fix the dark red heat at 640° C.* (1184° F.). Bauschinger observed the strength for puddled plate transverse to the direction of rolling at a dark red heat, to be 780 kil. against 2700 ordinary strength, and of rolled iron in the direction of the fibre at red heat, 750 against 4430 ordinary.

These results are important for the estimation of the strength of constructions which are exposed to the action of heat. Director Kirchweger, in Hanover, recognizes in the considerable decrease of strength at a red heat the sole cause of steam-boiler explosions, and endeavors to prove that

* "Ztschr. d. Vereins dtsch. Ing.," 1859, p. 265.

the boiler, even when filled with water, may be heated to redness.* Bauschinger considers it probable that in steam-boilers, the variations of temperature and differences between the outer and inner surfaces may affect the connection between the individual layers of the plate, and that thus the interior layers receive a disproportionate part of the total stress, while at the same time the shearing strength necessary for transference is diminished.† In this way, explosions certainly may occur. In the case of the exploded locomotive "Fugger," which had been twenty years in service, Bauschinger found the shearing strength between the layers less than in any of the other experiments.

A main point of the discussion at the present time is the influence of severe cold upon the resistance to rapid changes of strain, especially impact. It can hardly be denied that more axles and wheel-tires break in severe winters than in summer. While now, many seek the explanation of this and similar phenomena in the composition of the material itself, Styffe assumes that the rupture of metallic bodies is often caused by the fact that they are so constrained that they cannot yield to the contractile force due to cooling; moreover, axles, wheel-tires, rails, etc., are more easily broken in winter, because of the increased impact due to the diminished elasticity of the sleepers and of the ballast.

Such influences no one will deny; that they do not, however, concern the main point is shown clearly by Sandberg, inspector of railroad material for the Swedish Government, in his appendix to the English translation of the work of Styffe. Sandberg laid his rails upon granite blocks

* "Die Mühle," Jahrg., 1876, p. 24.

† "Zeitschr. d. bair. Arch. u. Ing.-Vereins," 1873, p. 47.

resting on each side upon the granite formation in the neighborhood of Stockholm, in such a manner that the elasticity of the foundation was the same both in winter and summer. The rails were tested in pairs; in winter at — 12° C. and in summer at + 29° (10° F. and 84° F.), by blows from a weight weighing 380 kil.* (837·75 pounds), and for the same rail only $\frac{11}{18}$ of the height of fall, as a mean, was necessary for rupture at — 12° that was required at + 29°. It was thus clearly proved that at least for certain kinds of iron, the resistance to impact is impaired by frost. Styffe has only made tests with dead loads, and in this respect his results are completely reliable.

Sandberg obtained further the peculiar result that rails from Aberdare, in Wales, which, in the heat of summer, sustained a height of fall 20 per cent greater than rails from Creuzot, showed in winter a resistance 30 per cent less than these. This can only be accounted for upon the assumption that the rails from Aberdare contained a larger amount of such material as gave a greater decrease of resistance to impact in winter than in summer. Fairbairn had already pointed out the prejudicial action of phosphorus and sulphur under severe cold, and thus Sandberg considers it probable that very different results might have been attained if the material had been entirely or at least approximately free from phosphorus. Unfortunately the chemical constitution of the pairs of rails was not determined. It appears, however, certain that phosphorus, which always diminishes the resistance to impact, exerts this influence in a special degree under severe cold. It is interesting to

* See the Tables of Sandberg in the Appendix to Styffe.

observe that this action of phosphorus increases also by severe heating. A screw-bolt of iron, rich in phosphorus, experienced, according to Styffe, such a change of structure by heating that a single stroke of the hammer sufficed to break it. Steel loses, with increasing percentage of phosphorus, the capability of sustaining repeated heating without losing its characteristic properties.*

In the year 1871, four treatises were presented to the Manchester Literary and Scientific Society by Joule, Fairbairn, Spence, and Brockbank, upon the influence of cold on iron and steel.† All the authors agreed that the strength for dead load was not diminished by cold, but rather, if any thing, increased. With reference to the resistance to impact, Brockbank attributed, as beyond doubt, to cold a considerable diminution, while Joule and Fairbairn would not admit such an influence. All of the authors call experiments to their support. The precision of Joule's experiments no one certainly will call in question; they do substantiate in some degree the action in question, but since the specimens were wires, darning-needles, and cast-iron nails, the results cannot be taken as holding for the various kinds of iron in use. Fairbairn and Spence, on the other hand, experimented only with a dead load. A number of the results of Brockbank perfectly confirm those of Sandberg. Thus, rails were tested by impact, which resisted much less when cold than at the ordinary temperature; a hollow cast-iron core about which a cylinder had been cast and cooled to $-7\frac{1}{2}°$ C. (18° F.) broke off as it was twisted

* Styffe, pp. 50, 95.

† See the "Bericht" in "Engineering," 1871, p. 82; also, "Ztschr. d. v. dtsch. Ing.," 1871, p. 775; "Polyt. Centralblatt," 1871, p. 476.

out, and showed a brittle fracture, while the pieces after heating were again perfectly strong and dense; an ice-covered rod of round iron of best quality, of 38 mm. diameter (1·5 inches), submitted to a freezing temperature for a week, at a temperature of 4½° C. (40° F.), broke off short at a single blow of a hammer of 5·4 kil. in weight (15·4 pounds).

All authors agree upon an increase of the tensile strength by cold, even although they may deny any diminution of the resistance to impact. This is the most unfortunate position imaginable. The strength for dead load is, to be sure, somewhat increased by cold, and so also, according to Styffe, is the elastic limit, just as is the case by hammering, rolling, tempering, etc. Since, however, in all these last cases the resistance to impact undeniably decreases, there is at least no reasonable ground to conclude that the contrary holds good in the first case. Even Styffe admits that iron is stiffer at low temperatures, thus confirming Sandberg, who found the permanent deflection for rupture noticeably less in winter than in summer.* But this is a property which in general goes hand in hand with a decrease of resistance to impact.

Thurston concludes from his own experiments,† that phosphorus and other impurities which produce cold shortness, may, under severe cold, influence the resistance to impact, but that this is the exception, and that generally the strength *for impact* as well as for a dead load is increased by cold. This, indeed, is new, but must first be established. Thurston's testing-machine† is, in general, well suited, by

* Styffe, pp. 129, 143.

† Dingler's "Polyt. Journ.," 1875, Vol. 217, p. 357. See APPENDIX for correction of errors of interpretation by Weyrauch.—TRANS.

reason of its convenience and cheapness, for demonstrations in the lecture-room, but not necessarily for scientific experiments, for which exact numerical determinations are necessary. The velocity applied has a certain influence, and is not regulated; the appliances for measurement are much too primitive to take account of such small differences as the influence of temperature; and, finally, we are by no means obliged to assume that experiments upon torsion are the most suitable for establishing the strength properties of fibrous and laminated materials.

In a report by Thurston * to the Massachusetts Railroad Commissioners in 1874, upon the cause of rail fracture, it is also stated "that iron and steel exposed to cold are not rendered brittle or unreliable for mechanical purposes, and that it is by no means the rule that the most failures occur on the coldest days." The composition of the commission is not given; it is also not quite clear upon what material the experiments or observations were made. Was it rich in phosphorus? Were the rails iron or steel? In northern latitudes, such as Canada, Sweden, and Russia, it has been observed that a soft steel containing from $\frac{1}{8}$ to $\frac{1}{2}$ of one per cent carbon is affected much less by cold than iron; but according to Styffe, there is no example or warrant for a well-recognized steel containing more than 0·04 per cent of phosphorus; for an English iron rail, however, there was 0·25 per cent, and for Dudley iron 0·35.

Upon a review of all the preceding, we can only come to the following conclusions: (*a*) Iron and steel which are completely or approximately free from foreign elements,†

* [Quoted by Thurston. *Vide* "Jour. Franklin Institute," 1874; "Dingler's Polytechnisches Journal," 1875–6.]

† By "foreign elements" we understand here every thing except carbon.

for practically occurring temperatures, neither gain essentially in strength for a dead load, nor lose noticeably their resistance to impact. (*b*) Certain elements which are not yet satisfactorily determined, but to which in any case phosphorus belongs, may, according to the extent to which they enter, cause a very noticeable decrease of the resistance to impact and rapid changes of strain. (*c*) The question can only be definitely settled by special experiment, when in each case the chemical composition of the material tested is determined. (*d*) In order to come to a conclusion, we must also have statistical data upon rupture in cold and warm countries, in warm and cold seasons, after long frost which hardens the ground, and in days of sudden, intense cold; but here also data upon the composition must not be omitted, and the notice of special forms and methods of production is of interest.

All the preceding, excepting a remark upon the steam-boiler, refers to the immediate influence of the temperature. With regard to the final action of *change* of temperature, Wöhler has propounded the view "that the repeated motion of the molecules by heat acts to destroy the connection of the parts in the same manner as the vibrations caused by outer forces." Experimental data upon this point do not exist. Spangenberg, according to his view of the phenomena of rupture, will not admit such an influence. Bauschinger considers it probable, from experiments with boiler-plate, that the strength of plate is diminished by the continued action of heat, but adds that this in no case amounts to

much. No account is here taken of *repeated* action, and it is not certain that the loss of strength may not be occasioned by burning or by the action of foreign substances during the heating.

If we accept the hypothesis of Wöhler, we have, in variation of temperature, a source of disturbance not only for metals, but also for all solid bodies. In this case, coefficients of safety avail nothing, for, if in any case we make a bar twice as strong as in another, still in the first case each half would be subjected to as much strain as in the second. However it may be, our bridges and structures which are subjected to relatively small temperature variations, would certainly have failed in some other manner, even if the changes of temperature had exercised a prejudicial influence.

CHAPTER XI.

MISCELLANEOUS RESULTS.

BAUSCHINGER found for steel the tensile carrying strength for flexure, that is, the greatest fibre strain at the moment of rupture, in accordance with the ordinary theory of flexure, always greater than the absolute tensile stress (compare table in Chapter IX.). Wöhler obtained the same result for iron and steel, but the primitive safe strength was for flexure not less than the tensile stress. The experiments of Styffe and Bauschinger have shown that the coefficient of elasticity for flexure may, without great error, be taken the same as for tension. All these observations indicate that the customary theory of flexure gives practically satisfactory results. Of especial interest in this connection are some experiments by Bauschinger, in which the length of the neutral axis or elastic line remained actually unchanged, and the originally plane sections remained perpendicular to it even for very considerable deflections.* If, therefore, we cannot insist that the method of calculation, especially as applied to thin-webbed plate-girders, is in every particular exact, still it covers the case quite as well as the method of calculating jointed structures, by which the apices are assumed to be hinged, although in reality they may be riveted, and is more correct than the ordinary methods of calculating compound systems of frame-work.

* "Ztschr. d. bair. Arch. u. Ing.-Vereins," 1873, p. 87.

The coefficient of elasticity for steel is found to be as a mean per square centimetre, by

Experiments upon flexure by Kupffer, 2124990 kil. (*cast-steel and file steel*).
" upon tension and flexure by Styffe, 241300 (*Bessemer steel*).
" upon tension by Bauschinger, 2215000,
" upon compression by Bauschinger, 2391000,
" upon flexure by Bauschinger, 2110000,
} *Bessemer steel specimens, especially prepared for experiment.*
" upon crushing by Bauschinger, 2082500 (*Bessemer round bars*).
" upon tension by Bauschinger, 2310000 (*Bessemer tires*).

Bauschinger found also from experiments upon shear and torsion, 862000. From all these experiments, we are justified in taking as a mean value for steel:

For tension, compression, flexure, and crushing, $E = 2150000$.

For shear and torsion (compare Art. 15), $E' = \frac{2}{5} E = 860000$.

Kupffer found from experiments upon English tire-iron, bar-iron, and Swedish wrought-iron, a mean value of 2053070. Styffe gives for good iron, containing very little phosphorus, 2171100, and for iron rich in phosphorus and slag, 1930600. These experiments confirm the already customary mean values, namely:

For tension, compression, flexure, and crushing, $E = 2000000$.

For shear and torsion, $E' = \frac{2}{5} E = 800000$.

No influence of carbon upon the coefficient of elasticity can be traced, but, according to Styffe, and also Kupffer, the latter appears to increase somewhat with the specific weight and with decrease of temperature.* A decrease is caused,

* Styffe, p. 122; upon page 77 the opposite is recorded. Wüllner, Experimentalphys., Vol. I., 1874, p. 204.

according to Tresca and Styffe, by transgression of the elastic limits, as also by mechanical treatment, when cold. According to Kupffer also, tempering may cause in hard steel a diminution of the coefficient of elasticity of about 6·5 per cent, while inversely Morin ascribes to cast-steel a possible increase of E by tempering of about 50 per cent. The specific weight diminishes somewhat, according to Wertheim's theoretical views and Kirkaldy's experiments, when the metal is worked cold, or, in general, when it is strained beyond the elastic limits, while at the same time the volume does not diminish as is so generally supposed.* All these influences are, however, not so great, and in part also not so irregular, as to demand a general consideration. In the calculation of weights for the purpose of estimates, the specific weight of wrought-iron may be taken at 7·6 to 7·7; that of steel at 7·8.

* Wertheim, in Poggendorff Bd. LXXVIII.; Wüllner, Experimentalphysik, Vol. I., 1874, p. 177.

CHAPTER XII.

ESTIMATION OF MATERIAL.

A BODY will sustain a greater stress, without any perceptible permanent change of form, the higher the elastic limits. If, therefore, we increase the last by hammering or tempering, the body will return from greater extensions or deflections to its original condition. Extended application is made of this in springs. If the ordinary elastic limits held good for all kinds of stress, and if we were certain that they could never be exceeded, it would be of advantage, in every construction, to make the elastic limits as high as possible.

The usual elastic limits, however, no longer hold good when we have to do with sudden stresses, or especially with impact; and even where they do hold good, we are by no means certain that they may never be exceeded. Thus, for example, in our riveted bridges, a stress not uniformly applied may readily cause local transgression of the limits. This is less injurious if the material possesses, even beyond the elastic limits, sufficient resistance. There may then occur such a gradual change of form as shall correspond to an approximately uniform distribution of the stress upon the entire cross-section.

We may exceed the elastic limits to a greater extent, and more frequently, the more ductile and tenacious the material. It is, moreover, a universally recognized fact that a material of great ductility and tenacity, of the same carrying strength, better resists impact and rapid changes of strain.

The choice of material, therefore, for most constructions, depends not only upon the strength or the elastic limit, but also very essentially upon the ductility and tenacity. (The union of tenacity with ductility constitutes "toughness.")

The greater the degree of the last-named properties, the more a bar will elongate up to rupture under the action of a permanent stress, and the greater also the diminution of the cross-section at the point of rupture—assuming, of course, that the conditions of the specimen and of the stress are the same.

The contraction at the point where rupture occurs takes place shortly before rupture. Corresponding to it there is a considerable local elongation, which is independent of that extension always connected with transgression of the elastic limits, and proportional to the length of the piece.

Since thus the total elongation for rupture consists of two parts, of which one is almost independent of the length, and the other is approximately proportional to the length, the ratio $\frac{\lambda}{l}$ of the total increase of length to the original length, can only be taken as a measure of greater or less ductility in the case of pieces of equal length; because the shorter the piece, so much the greater, relatively, is the local extension at the place of rupture to the total elongation.

Kirkaldy, who has studied closely, by numerous experiments, all the circumstances, insists that the contraction of the ruptured area must be considered as well as the amount of stress, in the estimation of the quality of the material. The stress upon the unit of area of the contracted cross-section of rupture increases with both, and this stress gives the best means of estimating the quality of the material. If

we arrange the various materials according to the values thus found, we obtain a useful comparison, while by ranking them according to the carrying strength alone, very ordinary or even inferior kinds may stand first,* for the simple strength may, according to Kirkaldy's experiments, be quite large for coarse-grained crystalline kinds, as well as for very dense and tenacious materials.

The mechanical working and the method of production in general, exert, as we have seen, an influence upon the strength as well as upon the tenacity and ductility. Thus iron plate in general has a less carrying strength than round iron. The same holds good for the ductility. That, therefore, which in one material is low may in another, by reason of the method of production, range high. With reference to the proposition and experiments of Kirkaldy, the Department of Public Works in India has published the following table to apply to the various kinds of iron in contracts and proposals :†

KIND OF MATERIAL.	CLASS C.		CLASS D.		CLASS E.		CLASS F.		CLASS G.	
	t for tension.	Contraction.	t for tension.	Contraction.	t for tension.	Contraction.	t for tension.	Contraction.	t for tension.	Contraction.
Round and square iron..	4250	45	4092	35	3937	30	3775	25	3620	20
Flat iron............	4092	40	3937	30	3775	25	3620	20	3466	16
Angle and T iron.......	3937	30	3775	22	3620	18	3466	15	3300	12
Plate across..	3466	12	3150	9	3000	7	2830	5	2675	3
Plate mean............	3620	16	3375	12	3233	9.5	3065	7.5	2912	5.5

* Compare the tables of v. Kaven, "Ztschr. d. Hannöv. Arch. u. Ing.-Vereins," 1868, pp. 443–446.

† "Deutsche Industriezeitung," 1873, p. 185.

The conditions for Classes A and B were given for certain cases. The contraction is given in per cent of the original cross-section.

From these numbers, which are based upon very extensive experiments, we see, among other things, that it is advantageous to use for the tension members of our bridges flat iron as much as possible. For roof-trusses, round iron is very suitable.

In America, the conditions for bridge proposals demand in general a higher carrying strength (from 3900 to 4200 kil. usually); a proof of the tenacity is also required by the, from the above not entirely unobjectionable test, that the specimen shall elongate from ten to fifteen per cent of its length before it ruptures. In order, upon the other hand, to avoid too soft a material, elastic limits are prescribed from 1600 to 1750 kil. The uniformity of the coefficient of elasticity is also regarded—as, for example, for the recent bridge over the Ohio River, the specifications allowed no deviation of over ten per cent in the experimentally determined coefficients of elasticity. Besides these preliminary tests, each individual tension-piece in the construction was tested up to double of its allowed stress; that is, up to about 1400 kil. per sq. centimetre, and it was frequently required that the piece when under this stress should also resist a sharp blow of the hammer. In Europe, it is generally thought that the Americans allow considerably greater strains in their bridges, but, as with us, the usual value of b is about 700 kil. per sq. centimetre* (10,000 lbs. per sq. inch).

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* These data are taken from an article by Gleim, "Der amerik. Brückenbau der Neuzeit," Ztschr. d. Hannöv. Arch. u. Ing.-Vereins, 1876, p. 73.

The fact that for the same carrying strength the primitive safe strength is greater the greater the ductility, is evident, no matter whether u lies above or below the ordinary elastic limits. This last case must be admitted, in view of the experiments of Tresca, already cited, in which the elastic limits were extended nearly up to t; in view also of the facts, that beyond the elastic limits permanent changes of form occur, and that generally for many kinds of stress the ordinary elastic limits possess no particular significance. It is by no means improbable that in time a sufficiently determinate relation may be established between the primitive safe strength on one side, and the stress per unit of area of the ruptured cross-section on the other, or generally between u and the directly observed values in experiments with permanent load, so that u may be readily determined, at least approximately, for each material, and thus the special numerical value for Launhardt's formula given. Then, perhaps, the vibration strength might be established, or at least estimated, by some relation. For such different materials as Phœnix iron and Krupp cast-steel, Wöhler found the ratio $\frac{s}{u}$ almost the same—namely, $\frac{7}{12}$ and $\frac{8}{15}$. It would be well also to make with materials, for which t, u and s have been determined by extensive experiments, as many simple tests as possible (such as flexure, impact, etc.); we should thus obtain a somewhat better scale of estimation than is furnished by those tests which thus far manufacturers have been accustomed to make in their proposals.

CHAPTER XIII.

ALLOWABLE STRESS PER SQUARE CENTIMETRE.

WHEN, by the statical calculation, the various stresses are determined, we have in the working strength a, that strain per square unit which for the greatest possible number of repetitions is still less than that necessary for rupture. No notice is by this taken of such prejudicial influences as do not occur in a systematic manner, such as impact, shocks due to traffic, flaws in the material, etc. These can only be embraced in the calculation by suitable coefficients of safety, and thus the allowable stress b is to be taken a certain fraction of the working strength a.

A.—WROUGHT-IRON.

TENSILE STRESS ALONE, OR COMPRESSION ALONE.

P 13

From formula I., the working strength is

$$a = u\left(1 + \frac{t - u}{u}\,\frac{min\,B}{max\,B}\right)$$

For axle iron from the Phœnix Co., Wöhler's experiments upon flexure give $t = 4020$, $u = 2195$, and therefore $\frac{t-u}{u} = \frac{5}{6}$, and the working strength for flexure

$$a = 2195\left(1 + \frac{5}{6}\,\frac{min\,B}{max\,B}\right)$$

In order, however, that our formula may not give for any stress too high a value, we must always consider the most unfavorable method of loading.

For the same iron under ordinary tensile stress, u was likewise 2195, but t was 3290. It appears from this, that such axle iron is a material whose strength can hardly be considered as sufficient in bridge construction. No greater strength should, however, be assumed, and we therefore have

$$\frac{t-u}{u} = \frac{3290-2195}{2195} = \tfrac{1}{2},$$

which gives for the working strength

$$a = 2100\left(1 + \tfrac{1}{2}\,\frac{min\ B}{max\ B}\right).$$

If, now, we take $\frac{1}{3}$ as the coefficient of safety, we have as the allowable stress per square centimetre

$$b = 700\left(1 + \tfrac{1}{2}\,\frac{min\ B}{max\ B}\right) \qquad (11)$$

In this expression $max\ B$ is the greatest and $min\ B$ the least stress which can come upon the piece in question.

ALTERNATE STRESS OF TENSION AND COMPRESSION.

We have from formula II., the working strength for this case

$$a = u\left(1 - \frac{u-s}{u}\,\frac{max\ B'}{max\ B}\right).$$

Wöhler found for Phœnix iron, $u = 2190$, $s = 1170$, and hence $\frac{u-s}{u} = \frac{7}{15}$, and

$$a = 2190\left(1 - \tfrac{7}{15}\,\frac{max\ B'}{max\ B}\right)$$

Taking again the lowest round numbers, this becomes *

$$a = 2100\left(1 - \tfrac{1}{2}\frac{max\ B'}{max\ B}\right)$$

and for a coefficient of safety of $\frac{1}{3}d$, we have the allowable stress

$$b = 700\left(1 - \tfrac{1}{2}\frac{max\ B'}{max\ B}\right) \qquad (12)$$

In this equation, the two opposite maximum stresses (*max* B' for the smallest and *max* B for the greatest) are to be inserted in their numerical values without regard to their signs or direction of action. For the determination of cross-sections, we shall in the following make use only of *max* B (Chapter XIV.).

A Few Special Cases.

For pieces which are permanently subjected to the same dead load, we have from (11), since *min* $B =$ *max* B,

$b = 1050$ kil. per sq. cent., or $b = 15{,}000$ lbs. per sq. inch.

For pieces which are always subjected to stresses of one character only, and return always to their original unstrained condition (as, for example, small plate-girders in which dead load is neglected), since *min* $B = 0$ in (11) or *max* $B' = 0$ in (12), we have

$b = 700$ kil. per sq. cent., or 10,000 lbs. per sq. inch.

For the flanges of bridges and roof-trusses, if p is the

* By putting $\frac{1}{2}$ in place of $\frac{9}{19}$ we also take the lowest round number, because we subtract too much or b is diminished. To obtain b in pounds per square inch, put 10,000 in place of 700.

dead weight and q the total weight per running metre, then, since, $min\, B : max\, B :: p : q$, we have from (11)

$$b = 700\left(1 + \tfrac{1}{2}\frac{p}{q}\right) \qquad (13$$

For pieces in which the maximum tensile and compressive stresses are equal, we have from (12), since then, $max\, B' = max\, B$,

$$b = 350 \text{ kil. } (5000 \text{ lbs. per sq. in.})$$

B. Steel.

STRESS OF TENSION ALONE OR COMPRESSION ALONE.

For Krupp cast-steel, Wöhler found $t = 7340$, $u = 3510$; hence $\dfrac{t-u}{u} = \frac{7}{6}$ and

$$a = 3510\left(1 + \tfrac{7}{6}\frac{min\, B}{max\, B}\right)$$

Since, however, the variations of strength for steel are considerable, and in bridge construction caution is in place, we take

$$a = 3300\left(1 + \tfrac{9}{11}\frac{min\, B}{max\, B}\right)$$

and hence obtain for a coefficient of safety of $\frac{1}{3}$

$$b = 1100\left(1 + \tfrac{9}{11}\frac{min\, B}{max\, B}\right) \qquad (14)$$

This formula gives threefold security, when $t = 6000$ and $u = 3390$ kil. The carrying strength of 600 corresponds, according to formula (9), to a steel of about 0·6 per cent carbon, which may be recommended for bridges. The primitive safe strength u, according to Wöhler's experi-

ments with cast-steel axles from Krupp, Bochum, and Seebohm, and cast-steel plate from Krupp, lay between 3300 and 3500; for cast spring steel (untempered) from Mayr in Leoben, and from Krupp, it was 3650. As a matter of course, only the best material should be used for bridges, and thus, for example, not more than 0·03 per cent phosphorus is allowable.

ALTERNATE TENSILE AND COMPRESSIVE STRESS.

For the same cast axle steel (furnished in 1862), Wöhler found $s = 2050$; for $u = 3510$, therefore, we have $\frac{u-s}{u} = \frac{5}{12}$ and

$$a = 3510\left(1 - \frac{5}{12}\,\frac{max\,B'}{max\,B}\right)$$

If we take then

$$a = 3300\left(1 - \frac{5}{11}\,\frac{max\,B'}{max\,B}\right)$$

and assume a coefficient of safety of $\frac{1}{3}$, we find for the allowable stress per square centimetre

$$b = 1100\left(1 - \frac{5}{11}\,\frac{max\,B'}{max\,B}\right) \qquad (15)$$

This formula gives still threefold security, if $u = 3300$, below which, according to Wöhler, the primitive safe strength for steel did not sink, and when $s = 1800$; while for cast axle steel from Krupp, Borsig, and Bochum, the vibration strength lay in the neighborhood of 2000.

SPECIAL CASES.

For pieces permanently loaded, we have from (14)

$$b = 2000 \text{ kil.}$$

For pieces which are subjected always to stress of one character, and then return to an unstrained condition, we have from (14) or (15)

$$b = 1100 \text{ kil.}$$

For the flanges of bridges and roof-trusses, and generally for pieces for which $min B : max B :: p : q$, we have from (14)

$$b = 1100\left(1 + \tfrac{9}{11}\frac{p}{q}\right) \qquad (16)$$

For pieces subjected to equal maximum stresses of tension and compression, we have from (15)

$$b = 600 \text{ kil.}$$

C. Remarks.

The coefficient of safety and the value of the allowable stress, as given above, for iron and steel, are chosen with special reference to bridge construction. The usual practice heretofore has been to take the allowable stress as constant, for wrought-iron at about 700 kil. per sq. cent. We see, however, from the preceding, that for a given factor of safety, b varies between 350 and 1050 (5000 lbs. to 15,000 lbs. per sq. in.), or from one to three times. The most favorable case is that of a dead load; that of tension alone or compression alone, the original condition being one of no strain, holds the mean place; and the most unfavorable case is that of alternate and equally great tension and compression. We see from this what very different factors of safety the old method of dimensioning gives to the various members of our bridges. Since, however, the least factor of safety at any point of a structure rules the whole, we have

thus far expended our material uselessly without obtaining thereby corresponding security. There is no sense whatever in allowing for sections subjected to stresses of from 700 to 1050 only 700, and then allowing also 700 in places where 350 is sufficient. If in a bridge there is just one diagonal or vertical which is subjected to approximately equal tensile and compressive stress, then the safety of this bridge is only half that usually supposed. It is worthy of consideration whether such exceptionally weak places in already completed structures ought not to be strengthened, and thus the safety of the whole construction increased.

The above given values of b for wrought-iron give still threefold security, when $t = 3150$, $u = 2100$, and $s = 1050$. Wöhler considers it allowable for constructions of unlimited life, to take for alternate strainless condition and tension or compression only, 1100 (above 700), and for alternate equal tension and compression, 580 (above 350). These numbers, since the point of departure is the same as in the preceding, correspond to a coefficient of safety of $\frac{1}{2}$, and to a rather too optimistic view. For constructions which last only a certain number of years, and therefore have to resist only a relatively limited number of repetitions of stress, the values of u and s may be taken greater than those given above (Chapter III.). For the present, this may be taken into account, by simply taking a different coefficient of safety, and thus retaining the working strength as above. In such cases, under the most favorable conditions, we may take a coefficient of $\frac{1}{2}$.

I have not deduced the value of b for steel from Wöhler's experiments with Krupp's cast spring steel, which, as far as they go, are best suited for such use, because all the three

values, t, u, and s_r have not been determined for this material. This steel shows, moreover, properties which can only seldom be assumed. For bridges a softer and more ductile material should be used. Although the carrying strength thus diminishes, it does not follow that it diminishes in *the same* degree as the primitive safe strength, because this last depends also upon the ductility. Thus, for decreasing t for tempered cast-steel,* $\frac{u}{t}$ was found to be $\frac{1}{2.50}$, and for untempered $\frac{1}{2.20}$; for cast axle steel $\frac{1}{2.08}$, and for iron from $\frac{1}{1.83}$ to $\frac{1}{1.5}$. In the estimation of the working strength, therefore, attention must be paid to this. For Krupp cast spring steel the working strength is, according to Wöhler's experiments upon flexure,

$$a = 3650\left(1 + \tfrac{6}{5}\,\frac{min\,B}{max\,B}\right)$$

and for the same material, tempered,

$$a = 4390\left(1 + \tfrac{3}{2}\,\frac{min\,B}{max\,B}\right)$$

from which, by means of the for any case suitable coefficient of safety, we may easily determine the allowable stress b.

If for a steel bridge we wish to use very soft steel, it may be advisable to take the allowable stress still less. Thus if, for example, only 0·45 per cent carbon is desired and the

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* Where experiment fails, formula (5) is assumed to hold good, since it agrees with experiments so far as the latter have been extended.

maximum strength is fixed at about 5200 kil., we may have, instead of formulæ (14) and (15)

$$b = 1000\left(1 + \tfrac{3}{4}\frac{min\,B}{max\,B}\right) \qquad (14a)$$

$$b = 1000\left(1 - \tfrac{1}{2}\frac{max\,B'}{max\,B}\right) \qquad (15a)$$

These formulæ give for $t = 5200$, $u = 3000$, $s = 1500$ still threefold security.

In the Bessemer steel arch bridge in the *Champ de Mars*, 1000 kil. was taken as the allowable stress for all portions, whether extended or compressed. Smaller cast-steel bridges are to be found in Holland, and one of puddled steel in Sweden (railroad bridge over the Götha-Elf, 42 metres long, 1865–66). The most remarkable steel bridge since 1874 is the cast-steel braced arch bridge over the Mississippi at St. Louis, with a middle span of 158·5 metres (520 ft.), and two side spans of 152·4 metres (500 ft.). Steel bridges are to be preferred, especially because of the greater resistance under severe cold and in very large spans, because of the diminished weight. For medium and small spans, however, a very stiff system of construction is requisite. The difference of price between steel and iron bridges cannot, in view of the greater lightness of the first and the ever-increasing cheapness of steel, serve as an argument much longer.

CHAPTER XIV.

METHOD OF DETERMINING DIMENSIONS.

IF *max B* represents the absolute greatest stress of a piece in a structure, whether tension or compression, and b the allowable stress per square centimetre for this piece, then, in all cases, the necessary *acting* area (entire area minus the diminution due to rivet-holes, etc.) is,

$$F = \frac{max\ B}{b}$$

The new method of calculation differs, therefore, only with respect to the value of b.

In the following remarks, which, in view of the simplicity of the case, are indeed hardly necessary, we shall consider only tensile and compressive stresses. We shall speak of shearing stress further on.

A. FRAME-WORK IN GENERAL.

The statical determination of the stresses gives *min B* and *max B* for those pieces always submitted to stress of one character, as also *max B′ max B* for those which are exposed to alternate tension and compression. Thus, we have for wrought-iron, respectively:

$$b = 700\left(1 + \tfrac{1}{2}\frac{min\ B}{max\ B}\right) \qquad (11)$$

and

$$b = 700\left(1 - \tfrac{1}{2}\frac{max\ B'}{max\ B}\right) \qquad (12)$$

In the last expression, *max* B denotes the greater and *max* B' the least of the two maximum stresses of different character, which are to be inserted according to their numerical value, without regard to the signs which indicate their different character.

For the flanges of framed girders, we have for uniformly distributed load,

$$b = 700\left(1 + \tfrac{1}{2}\,\frac{p}{q}\right) \qquad (13)$$

where p is the dead weight and q the total load per unit of length.

For steel, we have, instead of the above formulæ, the equations 14, 15, and 16.

EXAMPLE.—Required to determine the allowable stresses and net area for all parts of the framed girder shown in Fig. 6. The statical calculation is explained by Ritter*, and gives the values shown in the figure (for the calculation of the rivets, see Chapter XIX.).

The statical calculation is based upon a uniformly distributed dead load of 1000, and a similar total load of 6000 kil. per apex. Hence for all parts of the flanges we have from (13)

$$b = 700\,(1 + \tfrac{1}{2}\cdot\tfrac{1}{6}) = 758 \text{ kil.}$$

or

$$F = \frac{max\ B}{758}$$

* Ritter, "Elem. Theorie u. Berech. eis. Dach- u. Brückenconstr.," 1873, p. 36. Ritter's method of statical calculation is especially convenient for the new method of dimensioning, because *both* limiting stresses are furnished in the simplest manner by a single equation.

For the vertical *VI*, we have from (11)

$$b = 700\left(1 + \tfrac{1}{2}\cdot\tfrac{1875}{15625}\right) = 742 \text{ kil.}$$

or

$$F = \tfrac{15625}{742} = 21.1 \text{ sq. cent. } (3\cdot 27 \text{ sq. in.}).$$

Again, for diagonal *IX*, we have from (12)

$$b = 700\left(1 - \tfrac{1}{2}\ \tfrac{4600}{9550}\right) = 531 \text{ kil.}$$

or

$$F = \tfrac{9550}{531} = 18.0 \text{ sq. cent. } (2.8 \text{ sq. in.}).$$

In the same way all the values for b and F for the pieces *II* to *X* are computed and given below. The values obtained by assuming, according to the usual practice, $b = 700$ kil. per sq. cent. (10,000 lbs. per sq. in.), are also given for comparison.

Pieces —	*I* (Flanges).	*II*,	*III*,	*IV*,	*V*,	*VI*,	*VII*,	*VIII*,	*IX*,	*X*,	
b =	758	758	758	758	742	742	688	688	531	531	kil.
F =	$\dfrac{max\ B}{758}$	31·7	39·2	27·7	29·8	21·1	22·4	15·8	18·0	12·7	sq. cent.

By the ordinary practice,

F =	$\dfrac{max\ B}{700}$	32·9	40·7	28·8	30·3	21·4	21·1	14·9	13·1	9·2	sq. cent.

For larger spans we find much greater relative differences (see the next example and further comparisons in Chapter XXX.).

B. Simple Plate-Girder.

If $max\ M_x$ is the greatest moment for any cross-section distant x from the left end, and if v is the distance of the outermost fibre from the neutral axis, we have, as is well

known, from the theory of flexure, the moment of inertia of the resisting area at x.

$$I = \frac{max\ M_x v}{b}$$

Ordinarily, however, the cross-section F of the flange is computed preliminarily by the approximate formula,

$$F = \frac{max\ M_x}{b\,h_0} - \tfrac{1}{6}\,d\,h_0$$

where h_0 is the distance between the centres of gravity of the flanges, and d is the thickness of the web. If we wish to take account of the weakening of the web by rivets, we must insert in $\frac{1}{6}\,d\,h_0$ only the effective part of h_0, which we may estimate in general as $\frac{3}{4}\,h_0$—other approximate formulæ are also in use. Whichever one is used, however, the determination of b is unaffected.

If for the determination of the moments we have given, a uniformly distributed dead load p, and a uniformly distributed total load q, we have for the whole girder,

$$b = 700\left(1 + \tfrac{1}{2}\frac{p}{q}\right) \qquad (13)$$

If, however, the calculation is made for concentrated wheel loads, we have a curve for the $max\ M_x$ and for the dead load alone a similar curve giving $min\ M_x$. We have then, for any cross-section x,

$$b = 700\left(1 + \tfrac{1}{2}\,\frac{min\ M_x}{max\ M_x}\right)$$

but here also b will be almost always constant.

Usually, the cross-section is made constant only for very

short girders, and in such case we can disregard the slight action of the dead weight, and put directly $b = 700$.*

Since b for the simple girder is thus always constant, we can evidently represent the cross-sections graphically just as formerly. For variable b, the same graphical method holds good which is given further on for the continuous girder.

EXAMPLE.—Assuming for bridges of the spans given below, the values of $\frac{p}{q}$ as given by Laissle and Schübler, and also given below;† required to find the allowable stress b, which holds good for the cross-section of plate-girders and for the flanges of framed girders.

We have here generally,

$$b = 700\left(1 + \tfrac{1}{2}\frac{p}{q}\right) \qquad (13)$$

and, for example, for $l = 40$ m.

$$b = 700\,(1 + \tfrac{1}{2}\cdot\tfrac{1}{8}) = 817 \text{ kil.}$$

In a precisely similar manner, we obtain the values in the third line of the following table:

$l =$	7	10	15	20	30	40	60	100 metres.
$\frac{p}{q} =$	$\frac{1}{8.3}$	$\frac{1}{6.2}$	$\frac{1}{4.7}$	$\frac{1}{4.2}$	$\frac{1}{3.5}$	$\frac{1}{3.0}$	$\frac{1}{2.4}$	$\frac{1}{1.9}$
$b =$	742	757	774	783	800	817	846	884 kil.

* Upon the calculation of plate-girders under the assumption of concentrated loads, see Weyrauch, "Allg. Theorie u. Berech. d. cont. u. einfachen Träger," Leipzig, Teubner, 1873; also, "Maximalmomente einfacher Träger bei festen und mobilen Lastsystemen," Ztschr. d. Hannöv. Arch. u. Ing. Ver., 1875, p. 467.

† Laissle and Schübler, "Der Bau der Brückenträger," I. Theil, 1869, p. 97.

We see that for long spans the allowable stress for equal safety increases considerably. It follows evidently that a long and heavy bridge is affected less by a moving load than a lighter construction. To this we may add, that in a long bridge the influence of impact is less, and good material and careful execution are more likely to be insisted upon.

C. Continuous Girder.

For the continuous girder, it is customary to represent graphically the curve for the maximum positive and negative moments. According, then, as these two maxima have for any cross-section x the same or opposite signs, we have respectively,

$$b = 700\left(1 + \frac{1}{2}\,\frac{min\ M_x}{max\ M_x}\right)$$

$$b = 700\left(1 - \frac{1}{2}\,\frac{max\ M'_x}{max\ M_x}\right)$$

In these expressions, $min\ M_x$ $max\ M'_x$ denote always the numerically smaller values of the maximum moments, without regard to sign.

For the determination of the cross-sections of the continuous girder, we may proceed as above indicated, according as it is a framed or plate girder. But the flange areas are usually computed by the formula,

$$F = \frac{max\ M_x}{b\ h_0}$$

in which, therefore, for plate-girders the resistance of the web is disregarded. With reference to the numerical calculation of dimensions, nothing further remains to be noticed.

It is usual, in the determination of the flange cross-section for the continuous girder, to proceed graphically. This has

heretofore been done by describing the curve for the absolute maximum moments M_s from the curves for the maximum positive and negative moments by revolution of the first about the axis of abscissas (Fig. 7). Since the distance h_0 between the flanges is or may be assumed constant, the curve for $max\ M_s$ gives to another scale the maximum stresses in the flanges directly;

$$max\, B = \frac{max\ M_s}{h_0}$$

and for a constant value of b the cross-section required is

$$F = \frac{max\ B}{b_0}$$

where $max\ B$ is to be determined by the method given in nearly every text-book.*

We may proceed now also in a method entirely similar. The only difference is, that instead of the curve for $max\ M_s$, we obtain in a very simple manner a curve for the reduced $max\ M_s$. Thus, since b is the actual allowable stress per sq. cent. at x, we have

$$F = \frac{max\ B}{b} = \frac{max\ B}{700} \cdot \frac{700}{b}$$

and therefore, in order to represent by a curve just as before the cross-section, we have to multiply the previous ordinates by $\frac{700}{b}$, or

$$\text{reduced } max\ M_s = \frac{700}{b} max\ M_s$$

The latter curve then need not be drawn.

* A statement which scarcely holds good for English literature. The number of text-books in English which satisfactorily treat the subject of the continuous girder form the exception and not the rule.—TRANS.

EXAMPLE.—A continuous girder has spans of 52, 65, 65, 52 metres. The statical calculation gave for the second span the curves represented in Fig. 7 for the negative and positive moments.* Required to draw the curve for the reduced $max\ M_x$ for this span.

To the values of x given in the figure we have the corresponding values of $max\ M_x$ and M'_x from the curves. Hence for $x = o$ we have

$$b = 700\left(1 + \tfrac{1}{2}\tfrac{482}{2587}\right) = 765 \text{ kil.}$$

$$\text{and red. } max\ M_x = \tfrac{700}{765}\, 2587 = 2367 \text{ m. kil.}$$

For $x = 33 \cdot 1$,

$$b = 700\left(1 - \tfrac{1}{2}\tfrac{230}{1823}\right) = 656 \text{ kil.}$$

$$\text{and red. } max\ M_x = \tfrac{700}{656}\, 1823 = 1945 \text{ m. kil.}$$

For $x = 51 \cdot 2$,

$$b = 700\left(1 - \tfrac{1}{2}\tfrac{721}{728}\right) = 354 \text{ kil.}$$

$$\text{and red. } max\ M_x = \tfrac{700}{354}\, 728 = 1440 \text{ m. kil.}$$

(1 metre kil. $= 7 \cdot 2$ ft. lbs.)

In a precisely similar manner, we have found the other values for b and red. $max\ M_x$ given in the following table, and can therefore draw the corresponding curve as shown in Fig. 7. The curve for $max\ M_x$, by which heretofore the cross-sections have been determined, is, for the purpose of comparison, also drawn in the figure.

* The complete calculation is to be found in Weyrauch, "Allg. Theorie u. Berech. d. cont. u. einfachen Träger," 1873, p. 113.

x	$max\ M_x$	$max\ M'_x$	b	red. $max\ M_x$
0	-2587	-482	765	2367
4	-1656	-330	770	1500
8·5	$-\ 970$	-0	700	970
14·1	$+\ 611$	-502	412	1038
23·0	$+1470$	-300	629	1636
33·1	$+1823$	-230	656	1945
42	$+1560$	-400	611	1787
51·1	$-\ 728$	$+721$	354	1440
57	-1275	$+179$	651	1371
59	-1570	0	700	1570
62	-2050	-220	737	1947
65	-2776	-390	751	2587

REMARKS.—In the statical calculation of the above girder, p was 2000 and $q = 2200 + 4500$. Assuming that the span of 65 m. is covered by a simple girder, and taking for this case $p = 2700$ and $q = 2700 + 4500 = 7200$ kil., we have for the curve of $max\ M_x$—that is, for the moment curve for total load,*

$$max\ M_x = \tfrac{1}{2} q\, x\, (l - x) = 3\cdot6\, x\, (65 - x)$$

For the flanges, b is constant and equal to

$$b = 700\,(1 + \tfrac{1}{2}\tfrac{2\cdot7}{7\cdot2}) = 851 \text{ kil.}$$

In Fig. 7, for the sake of comparison, we have given also this moment curve reduced to $b = 700$ kil. The ratio of the areas included by the shaded and dotted lines gives at a glance about the relative amount of material in the flanges for the continuous and simple girder respectively.

* Weyrauch, p. 87.

The saving of material by reason of continuity is thus somewhat less by the new metbod of dimensioning than formerly, but, on the other hand, the objection to the continuous girder, that certain pieces are subjected to alternate tension and compression, no longer carries weight, since we are now able to provide the same security for this kind of stress as for any other.

PART II.

SHEARING STRENGTH—RIVETING.

THE importance of carefully executed connections upon the strength and durability of iron constructions scarcely needs to be insisted upon. Nevertheless, in the arrangement and execution of riveting, by no means sufficient care is ordinarily shown. That grave disasters have not as yet more frequently occurred is due to the fact that there is a great friction between the parts drawn together by the nearly always hot executed work, caused by the contraction of the rivets in cooling. This friction ranges ordinarily between 800 and 1600 kil. per. sq. cent. of rivet area; it depends of course upon the length of the rivet, and at the beginning is generally sufficient, of itself alone, to resist the stress. But at the moment of rupture, for which there is a sliding of the plates and deformation of the rivet-heads, this friction can no longer be counted upon; it may also disappear in time by the action of vibrations, and then the rivets themselves come alone directly in play. It is, therefore, necessary to disregard this friction entirely in our calculations, and to rely solely upon the shearing strength of the material.

What, however, are we to understand by the shearing strength of the material? Heretofore it has been regarded as that force which, acting upon a unit of area, is just sufficient to cause rupture. But this is only the shearing force

for a once-applied permanent load, while, according to Wöhler's law, less stresses than this may cause rupture if repeated sufficiently often. Instinct teaches the same. If we try to break by hand the connection shown in Fig. 8 by shearing the bolt, we should perhaps first try by a single pull P; if this proves insufficient, we should repeatedly pull, and if then we do not succeed, might perhaps by alternate pulling and pushing attain our end. The strength, therefore, for shear is greatest for a permanent load, less for stresses of one character only, such that the material alternates between a strained and strainless condition, and least for alternating stresses in opposite directions, in which latter case also the friction is most readily overcome.

In the estimation of the strength of pieces exposed to tension and compression, it has been the custom, in many cases, to have regard to Wöhler's results, but not so for shearing strength. It avails, however, nothing to calculate the diagonals of a trussed girder for variable stresses, if for the rivets by which these very stresses are transmitted, only a permanent load is considered. If the rivets give way, it is a matter of indifference whether the diagonals fall with the bridge into the water in one or in two pieces.

Only by having regard to Wöhler's law can a greater safety than heretofore be attained, and a similar misapplication of material be avoided.*

* The general formulæ deduced for rivet connections hold good also for every other method of calculation; only the value of b is then changed.

CHAPTER XV.

CARRYING STRENGTH FOR SHEAR.

USUALLY only the ratio of the carrying strength for shear t' to that for tension t, is given (the last in the direction of the fibres). For the ratio $\frac{t'}{t}$ most authors take $\frac{4}{5}$ or 1. Those who take 1 admit, however, that this value is properly somewhat too great.

For a perfectly isotropic body, we have from the theory of elasticity*

$$\frac{t'}{t} = \frac{1}{1+n}$$

and for the ratio of the coefficients of elasticity

$$\frac{E'}{E} = \frac{1}{2(1+n)}$$

Navier, Poisson, and Clapeyron found theoretically that for all isotropic bodies $n = \frac{1}{4}$. In such case therefore

$$\frac{t'}{t} = \frac{4}{5}, \quad \frac{E'}{E} = \frac{2}{5}$$

Later investigations of Cauchy, Lamé, and Kirchoff show,

* See Winkler, "Die Lehre v. d. Elastig. u. Fest.," 1867, p. 46.

however, that we can only theoretically conclude that n lies between o and $\frac{1}{2}$; accordingly

$$\frac{t'}{t} \text{ between } 1 \text{ and } \frac{2}{3}$$

$$\frac{E'}{E} \text{ between } \frac{1}{2} \text{ and } \frac{1}{3}$$

Experiment must decide. The experiments of Kirchoff, Wertheim, Regnault, etc., made for the determination of n* have less value for materials used in quantities; they all give, however, n between $\frac{1}{4}$ and $\frac{1}{3}$ for iron and steel, so that, therefore, $\frac{t'}{t}$ lies between $\frac{4}{5}$ and $\frac{3}{4}$, and $\frac{E'}{E}$ between $\frac{2}{5}$ and $\frac{3}{8}$.

Wöhler showed under certain assumptions that $\frac{t'}{t} = \frac{4}{5}$ and $\frac{E'}{E} = \frac{2}{5}$ *when the shearing edges do not lie in one plane* † (Fig. 1). Corresponding experiments with bars sheared by permanent load gave $\frac{t'}{t}$ for axle iron of the Phœnix Co. $\frac{360}{445}$, for Krupp's cast-steel plate $\frac{560}{738}$, or in the last case approximately, in the first almost exactly $\frac{4}{5}$. For torsion $\frac{t'}{t}$ was found only little less, and $\frac{E'}{E} = \frac{1.95}{5}$. Bauschinger obtained exactly the same result, $\frac{1.95}{5}$ from a series of experiments upon torsion with Bessemer steel (compare Chap. 11). The ordinary conception of torsive strength as a kind of shearing strength seems therefore correct. Spangenberg ‡ drew also the

* Wüllner, "Experimentalphysik," Vol. I., 1874, Arts. 51–54.

† Wöhler, "Die Festigkeitsversuche mit Eisen und Stahl," 1870, p. 5.

‡ Spangenberg, "Ueber das Verhalten der Metalle," 1875, p. 20.

same conclusion from the appearance of fracture, and for the present we may assume the same coefficient of elasticity and the same maximum fibre strain as for shear. The experiments further confirm, with certain limitations, the general relations between tensile and shearing strength derived from the theory of elasticity.

That these relations can be made use of only with caution for bodies not isotropic, with which only we are concerned, is very plainly indicated by the experiments of Bauschinger.* The carrying strength for shear was found very different according to the position of the shearing plane with reference to the direction of the fibres. Bauschinger distinguished six different positions (Fig. 9). Of these, *I*, *III*, and *IV* are those practically important; *V* and *VI* do not in general occur. For the positions *I*, *III*, *IV*, t' was generally less than t for tension in the direction of the laminæ, for *II* the difference was considerably greater, for *V* and *VI*, as a mean $t' = \frac{1}{2} t$, and went as low as $\frac{1}{3} t$. The differences between t' for the various positions, were, as was to be expected, greater the more pronounced the fibrous and laminated character. We give a few results, especially for the positions *I*, *III*, and *IV*.

For rolled iron from *Wasseralfingen*, for original value of t of 3893,† the value of t' was for the position *I*, 3448; *II*, 2836; *III*, 3590; *IV*, 3060; *V*, 1787; *VI*, 1767; hence as a mean of *I*, *II*,

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* Bauschinger, "Versuche über die Zugfest. u. Schubfest. von Kesselblech u. Walzeisen." Ztschr. d. bair. Arch.- u. Ing.-Vereins, 1873.

† The specimens were the remnants after rupture by tension. Since the tensile strength was thus raised by the transgression of the elastic limits from 3893 to 4423, it is not certain that t' also was not increased. In case of a direct relation, this must be the case.

and *III*, $3336 = 0{\cdot}85\ t$. For two angle irons from the *Lothringer* works, t was 3160, and t' for *I*, 2630; *III*, 3030; *IV*, 2620, or $\frac{t'}{t}$ always therefore greater than $\frac{4}{5}$. For various German, French, and English boiler-plates, for a mean value of $t = 3180$, t' was found for *I*, 2410; *III*, 2460; *IV*, 2540, or general mean, 2470; almost exactly $\frac{4}{5}$ of 3180. For Styrian cast-steel plate for locomotive boilers, t' was found for *I*, 3920; *III*, 4380; *IV*, 4460, all mean values, t being 5025. Here, therefore, $\frac{t'}{t}$ was again greater than $\frac{4}{5}$.

To sum up, we may conclude that the best ratio $\frac{t'}{t}$ of the shearing to tensile strength in the direction of the rolling for the practical cases of *I*, *III*, *IV*, is $\frac{4}{5}$ as given already by Molinos and Pronier, Reuleaux, Gerber, and others, upon the ground of numerous experiments.* It may well happen, especially in boiler-plates, that t' for the positions *III* and *IV* may equal the tensile strength in direction of rolling, and may even be still greater, but it does not follow that this may, without further evidence, be assumed.

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* The Americans take $\frac{t'}{t} = \frac{3}{4}$. See Gleim, "Der Amer. Brückenbau der Neuzeit," Ztschr. d. Hannöv. Arch. u. Ing.-Vereins, 1876, p. 92.

CHAPTER XVI.

STRENGTH AND ALLOWABLE STRESS FOR SHEAR IN GENERAL.

FROM the experiments of Wöhler, in which the shearing surfaces lay in different planes (Fig. 1), and those of Bauschinger, in which these planes were consecutive, it appears that the ratio $\frac{t'}{t}$ does not depend in any recognizable way upon this difference. Wöhler further found, that for repeated stresses the strength for shear and torsion could be taken equal to $\frac{4}{5}$ of the tensile strength in direction of rolling, under similar conditions (that is, for equal ratios of the limiting strains), as was from theory to be expected. We shall, therefore, take this as true under the assumption that the plane of shear has one of the practically occurring positions *I, III, IV.* We then obtain for shear and tension, t', u', s', and generally the working strength a', by simply multiplying by $\frac{4}{5}$ the corresponding number t, u, s, a, for tension, and these hold good, under the assumption of the usual coefficients of safety, for the allowable stress per square centimetre.

For stress in one direction only, we have therefore from formula I.

$$a' = u'\left(1 + \frac{t' - u'}{u'}\,\frac{min\,B}{max\,B}\right) = \tfrac{4}{5}u\left(1 + \frac{t - u}{u}\,\frac{min\,B}{max\,B}\right) = \tfrac{4}{5}a \quad \text{(III.)}$$

This case applies to the connecting rivets of pieces (such as diagonals and verticals in bridges) which are always in

tension or compression, for the cross-sections of plate-girders in the vicinity of the supports, for the riveting of steam-boilers, etc.

The stress in alternating directions is from formula (II.)

$$a' = u'\left(1 - \frac{u' - s'}{u'} \frac{max\, B'}{max\, B}\right) = \tfrac{4}{5} u \left(1 - \frac{u - s}{u} \frac{max\, B'}{max\, B}\right) = \tfrac{4}{5} a \quad \text{(IV.)}$$

This case applies to the connecting rivets of pieces alternately extended and compressed, for cross-sections of plate-girders at the centre of the span, etc.

A. Wrought-Iron.

The allowable stress per square centimetre for the case that all stresses are of the same character is from (11),

$$b' = \tfrac{4}{5} b = 560 \left(1 + \tfrac{1}{2} \frac{min\, B}{max\, B}\right) \quad (18)$$

and for the case that the stresses are in opposite directions

$$b' = \tfrac{4}{5} b = 560 \left(1 - \tfrac{1}{2} \frac{max\, B'}{max\, B}\right) \quad (19)$$

The ratio $\frac{min\, B}{max\, B}$ is that of the smallest to the greatest shear in the same direction, and $\frac{max\, B'}{max\, B}$ is that of the two maximum forces in opposite directions (*max* B' the least, *max* B the greatest), only the numerical values to be used, without reference to the sign denoting direction of action. If we have to do with torsion, we take the same values for that case, and (18) and (19) give the allowable strain in the outermost fibre. We have from (18) for the special case of permanent load, since then $min\ B = max\ B$, $b' = 840$; for

pieces which, after each stress in the same direction, return to an unstrained condition, $b' = 560$; for pieces which are equally strained in opposite directions alternately, from (19) $b' = 280$.

B. Steel.

For stress in only one direction, we have from (14)

$$b' = \tfrac{4}{5}b = 880\left(1 + \tfrac{9}{11}\frac{min\,B}{max\,B}\right) \qquad (20)$$

For stress in alternate opposite directions, from (15)

$$b' = \tfrac{4}{5}b = 880\left(1 - \tfrac{5}{11}\frac{max\,B'}{max\,B}\right) \qquad (21)$$

For the special case of permanent load, $b' = 1600$; for pieces which return after each stress of always the same character to an unstrained condition, $b' = 880$; for pieces which undergo alternate stress in opposite directions, $b' = 480$ kil. per sq. cent.

C. Remarks.

One may, perhaps, regard the relation for shear and torsion in opposite directions as rather forced, but it is precisely this of which Wöhler's experiments give striking proof. Thus for Krupp cast axle steel u and s (3510 and 2050) as also u' and s' (2780 and 1610) were determined, and we have, both directly and indirectly,

$$a' = u'\left(1 - \frac{u' - s}{u'}\,\frac{max\,B'}{max\,B}\right) = 2780\left(1 - 0{\cdot}42\,\frac{max\,B'}{max\,B}\right)$$

$$a' = \tfrac{4}{5}u\left(1 - \frac{u - s}{u}\,\frac{max\,B'}{max\,B}\right) = 2808\left(1 - 0{\cdot}42\,\frac{max\,B'}{max\,B}\right)$$

If it concerns so soft a steel as is assumed in (14a) (15a) Art. 13, we may put instead of (20), (21)

$$b' = \tfrac{4}{5}b = 800\left(1 + \tfrac{3}{4}\,\frac{min\,B}{max\,B}\right) \qquad (20a)$$

$$b' = \tfrac{4}{5}b = 800\left(1 - \tfrac{1}{2}\,\frac{max\,B'}{max\,B}\right) \qquad (21a)$$

If the allowable stress for position *II* (Fig. 9) is concerned, we may put $b' = \tfrac{3}{4}\,b$, while for the practically useless positions *V*, *VI*, we must not take b' over $\tfrac{1}{2}\,b$, understanding always by b the allowable tensile stress for equal ratios of the limiting strains.

CHAPTER XVII.

WEBBING IN PLATE-BEAM.

FOR the thickness δ of the vertical web of plate-beams (Fig. 41), the calculation in general gives too small a value. Experiments have long shown that the plate-girder fails first by side-buckling; the forces thus arising elude any systematic investigation. It is sought to meet this difficulty, especially in large girders, by stiffening, but it is also advisable not to choose the thickness of the web too small. In this respect also it is to be borne in mind that the resistance of the plate-web is often weakened by rust, and that the pressure upon the rivet area should not be disproportionately great (Art. 24).

In the numerous plate-girders of the new Berlin transport road, the thickness of one centimetre (0·4 inch) is always observed, as well for the smallest rail-girders as for large main girders, and in any case the minimum allowable is 0·8, even if the following formulæ, which offer only a one-sided point of view, give a less thickness.

Beside the side-buckling, we have also tension, compression, and shear in the web. The horizontal tensile and compressive stress is proportional to the distance from the neutral axis, therefore less at every point of the web than in the outermost fibres of the girder cross-section, and therefore

less than the allowable. The horizontal shear per unit of height upon the entire thickness, which we may call the *specific* horizontal shear, is greatest at the axis, or *

$$H_0 = \frac{max\ V_x}{h}$$

where h is the distance between the centres of action of tension and compression. In order, then, to withstand the horizontal shear at every distance from the axis, a plate is sufficient, whose thickness δ is given by

$$1 \cdot \delta \cdot \tfrac{4}{5} b = \frac{max\ V_x}{h} \qquad \text{or}$$

$$\delta = \tfrac{5}{4} \frac{max\ V_x}{h\,b}$$

This plate would suffice also for the vertical shear, because at every point the specific vertical shear is equal to the horizontal. If we put here, where it is not generally specified, $h = \frac{9}{10} h_0$, understanding by h_0 the distance between the centres of gravity of the flanges, we have

$$\delta = \tfrac{25}{18} \frac{max\ V_x}{b\,h_0}$$

The inclined strains in the web can then be greater at any point x than the horizontal and vertical, when at this point, and *simultaneously*, the vertical shear V_x and the moment M_x become greater, as for the piers of continuous girders and small simple girders under concentrated loading. Still,

* Weyrauch, "Theorie u. Berech. d. Träger, 1873," p. 14.

even here only the inclined tension or compression takes a greater value, the greatest being at the beginning of the flange. From the practical investigation of a number of unfavorable cases, we may take account of the inclined strains, by putting for *max* V_x for the simple girder $\frac{10}{9}$ and for the continuous $\frac{5}{4}$ of this value, so that for each case we have respectively,

$$\delta = \frac{3}{2}\,\frac{max\ V_x}{b\,h_0} \qquad (22)$$

$$\delta = \frac{5}{3}\,\frac{max\ V_x}{b\,h_0} \qquad (23)$$

The last formula is given also by Laissle and Schübler.

According as at x, the shear V_x acts always in the same direction, as at the supports, or in opposite directions, as in the middle of the span, we have for b for wrought-iron*

$$b = 700\left(1 + \frac{1}{2}\,\frac{min\ V_x}{max\ V_x}\right)$$

$$b = 700\left(1 - \frac{1}{2}\,\frac{max\ V_x'}{max\ V_x}\right)$$

We have, however, in (22) (23) only the greatest value of $\frac{max\ V_x}{b}$, and this occurs, as we may easily assure ourselves by numerical calculation, for the simple and continuous girder, always at a support, so that we need to use only the *first* value of b.

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* For steel the corresponding formulæ are easily found from Art. 13. It would be superfluous to repeat them here.

If we have to consider a *simple plate-girder* for a *uniformly distributed load*, we have at the supports

$$max\ V_x = \tfrac{1}{2} q\, l \qquad min\ V_x = \tfrac{1}{2} p\, l$$

and therefore

$$b = 700\left(1 + \tfrac{1}{2}\frac{p}{q}\right)$$

or by substitution in (22)

$$\delta = \frac{q\, l}{930\left(1 + \frac{p}{2\, q}\right) h_o} \qquad (22a)$$

where p is the dead weight, q the total load in kil. per running metre, l the span in metres, and h_o the distance between the centres of flanges in centimetres, δ being likewise expressed in centimetres. (Substitute 13300 in place of 930, and δ will be given in inches, p and q being taken in lbs. per ft., l in feet, and h_o in inches.)

EXAMPLE.—For two simple plate-girders of $l = 7$ and 10 metres, $h_o = 70$ and 100 cent. Let $p = 900$, $q = 8100$, and $p = 1100$, $q = 6600$ kil. respectively. What should be the thickness of the web?

From (22*a*) we have at once,

$$\text{for } l = 7 \text{ m.} \quad \delta = \frac{8100 \times 7}{930\,(1 + \frac{1}{18})\, 70} = 0 \cdot 82 \text{ cent.}$$

$$\text{for } l = 10 \text{ m.} \quad \delta = \frac{6600 \times 10}{930\,(1 + \frac{1}{12})\, 100} = 0 \cdot 66 \text{ cent.}$$

Since for long girders the side flexure is more to be feared than for short ones, we may without inconsistency take for both the cases above $\delta = 0 \cdot 9$ to 1.

CHAPTER XVIII.

METHODS OF RIVETING.

VIEWS are still divided as to whether punched or drilled rivet-holes are best. Two points here must be considered: 1st, the probability of a good riveting; 2d, the strength of a good riveting. As regards the first, we may at once decide in favor of drilling, because the holes in the parts to be riveted better correspond. Sometimes indeed in punching, reaming is resorted to, but this is not generally available, else holes might arise not suited to the thickness of rivet.

The strength of the riveting depends upon (*a*) the strength of the punched or drilled plate, (*b*) upon the tenacity of the rivets. Every perforated plate must theoretically lose somewhat of its mean strength per square unit, at the rivet seam, for in Fig. 10 the tensile stress upon the middle portion can only be transmitted to the outer fibres by those about the hole, so that these last sustain a greater unit stress. Browne, who first called attention to this, observed also that the rupture of the perforated plate first began at the edges of the hole.*

The principal advocate of punching is Fairbairn; his principal argument, the test of quality thus furnished.† Fairbairn at every opportunity falls back upon the fact that poor iron tears in punching, and considers by this the case to be

* Walter Browne, "Punched and Drilled Rivet-Holes," Engineer, 1872, p. 362; Polyt. Centralblatt, 1873, p. 284.

† Compare the lectures of Fairbairn before the Royal Society, in "Engineer, 1873, p. 280; "Polyt. Centralblatt," 1873, p. 743.

settled. It is indeed very certain that poor iron, by the powerful action of punching, visibly tears; but that at least does not prove that other qualities do not suffer. From the following table, which contains the mean values of recent American experiments,* it seems, as might have been expected, that the best iron is weakened by punching; the difference which also exists in drilling is explained by the above theoretical condition.

Specimens 44 mm. wide; 8 mm. thick; diameter of hole 16 mm.	Method of rupture	Load per sq. cent. of plate cross-section.	Load per sq. cent. of rivet cross-section.
1. Entire plate...........	Plate torn.......	4200	
2. Plate with drilled hole.	Plate torn.......	3530	
3. Plate with punched hole	Plate torn.......	2690	
4. Single riveted, hole drilled............	Rivet shorn......	3280	3750
5. Single riveted, hole punched..........	Plate torn.......	3510	4030

The carrying strength for tension per sq. centimetre was also for the cross-section at the rivet seam, for drilled hole, 0·84, for punched only 0·64 of the unperforated plate—that is, for punched only 0·76 as much as for drilled. Experiments by Sharpe with Bessemer plate gave the strength in the first case even only 0·59 as great as in the last, and the ratio was, in general, the more unfavorable for punched holes, the more brittle the material, the thicker the plate, and the less the ratio $\frac{e}{d}$ of the breadth of piece to diameter of rivet.

* The experiments were made by a commission of the American Railway Master Mechanics' Association, "Engineer," 1872, p. 362; "Polyt. Centralblatt," 1873, p. 284.

As to the tenacity of the *rivet* in drilled and punched holes, the case is different. Experiments 3 and 5 give the resistance of friction per sq. cent. of rivet cross-section $3510-2690=820$ kil., so that the rivet in case 5 still sustained the shear $4030-820=3210$. In case 4, if the friction in the two cases is taken equal, it would have been sheared by $3750-820=2930$ kil. The rivet in the drilled hole bore therefore about 9 per cent less than in the punched hole. In spite of this, it is, however, clear that the strength of the rivet iron cannot depend upon whether it is inserted in a drilled or punched hole. Notwithstanding all objections, the difference can only be ascribed to the sharper sides of the hole, which exercise a cutting action. This is indicated, for example, by the experiments of Sharpe, Kirkaldy, and others,* in which the same rivets in hard steel plate showed a much less resistance than in iron plate, and this is confirmed also by the experiments of Fairbairn. Thus, he observed that the rounding off of the edges in drilled holes gave an increase of strength of about 12 per cent, but in punched holes only $2\frac{3}{4}$ per cent. If we assume, in experiment 5, the 3210 kil. as the strength of the rivet in an unrounded punched hole (this stress is about $\frac{4}{5}$ of the tensile carrying strength of the very good iron plate used, and therefore the rivet by the tearing of the plate in case 5 must have been at the limit of its resistance); and then assume Fairbairn's values as holding good, we should have had by rounding off the edges,

For punched hole.	For drilled hole.
$3210 \times 1{\cdot}0275 = 3298$ kil.	$2930 \times 1{\cdot}12 = 3281$ kil.

or almost a perfect coincidence. Such a correspondence

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* See "Ztschr. d. V. dtsch. Ing.," 1865, p. 604.

need not in every case be expected, since the influence of the rounding off depends upon the brittleness of the material, thickness of the plate, quality of the punching-machine, etc.; but we see, however, that the above reason can account for the different tenacity of the rivet in the drilled and punched hole. If the lubricant used in drilling is incompletely removed, the friction will be less, and this small and temporary influence may thus often be subtracted from the account of the rivet strength.

While, therefore, the strength of the plate is considerably greater for drilled than punched holes, the tenacity of the rivet itself is not different so soon as the holes are rounded off at the edges. This rounding off and the corresponding counter-sinking of the rivet-head has also the advantage that thereby the area to be sheared is increased, and the resistance thus somewhat increased.

The experiments of Fairbairn nowhere conflict with these views. The following loads per square centimetre of rivet area of cross-section caused, among others, rupture:

Rivet-hole.	Load per square cent.	Riveting by	
Punched	3080	Machine.	Single shear.
Drilled	2920	Machine.	
Punched	3240	Hand.	
Drilled	3200	Hand.	
Drilled and rounded off	3190	Machine.	
Punched	6970	Hand.	Double shear.
Drilled	6170	Hand.	
Drilled and rounded off	7250	Hand.	

That Fairbairn, from these and other experiments, concluded a greater strength and advantage for punched holes, is accounted for by the fact that he left out of sight the very different effects upon the plate strength of drilling and punching; this influence apart we must certainly decide in favor of punched holes.

In general, then, drilled holes are to be strongly preferred to punched; but we do not need on that account to prohibit punched once for all. If circumstances render the latter necessary, proper precautions should be observed. The places to be punched should be at least blood-warm; all torn places are to be excluded; the coincidence of badly corresponding holes should be improved by reaming; in the vicinity of the holes, the plates should be annealed after punching, and this is the more important the more brittle the material, and therefore imperative with steel. Sharpe found that the strength of punched steel plates of 0·8 cm. thick was increased by annealing from 3300 to 5200 kil.;* for drilled plate it was, without annealing, 5600. The difference of 400 may be the loss of carrying strength, which is always caused in hammered and rolled iron and steel by annealing (Art. 7). Kirkaldy found that by tempering also, the strength of punched plates increased, and still more so than by annealing. We should make use of it rarely, however, as the resistance to shock is thereby greatly diminished.

Seven experiments by Fairbairn gave as a mean 7½ per cent greater strength for hand than machine riveting. Fairbairn ascribes this to the circumstance that in hand-riveting the rivet-iron in an already somewhat cooled condition, is somewhat hardened. In general, he declares in favor of

* Haswell, "Studien über Bessemer Stahl, Techn. Bl.," 1874, p. 116.

machine-riveting, because the holes are more perfectly filled, and loose rivets hardly ever occur.

Warm riveting has the advantage over cold, that greater friction occurs, and that, by reason of the better set, rust cannot so readily enter between the plates. If the total thickness of the parts to be riveted is over 10 cm. (3·9 inches), it is better not to rivet warm, as then the contracting force in cooling may cause rupture of the rivet-stem or injury to the head. In any case for stem lengths of over 15 cm. (5·9 inches), hot riveting is not allowable.* In such case, however, even cold riveting is hardly in place, and we may to better advantage make use of screw-bolts. According to Gerber, such bolts have a strength 6 to 8 per cent greater than hot riveting,† a fact which he attributes to the better filling of the holes; certainly the cutting and wedge action of the hole-edges are diminished.‡

Disregarding the questions of machine or hand and cold or warm riveting, whose solution depends upon circumstances, the result of our discussion is as follows:

Best riveting: holes drilled, edges rounded, rivets counter-sunk.

Good riveting: holes punched at blood heat, poorly corresponding holes reamed out, edges rounded, rivets counter-sunk, vicinity of the holes annealed. This last is only superfluous for very thin plates—such, for example, as occur in bridge construction.

* Upon the proper dimensions of rivets and practical requirements in riveting, information and literature references are given by Ludewig, "Ueber Vernietung," Ztschr. des Vereins deutscher Ingenieure, 1869, 1872.

† Gerber, "Berech. d. Brückentr. nach System Pauli," Ztschr. d. Ver. dtsch. Ing., 1865, p. 480.

‡ Compare Stoney, "Theory of Strains," Art. 424. London, 1869.

CHAPTER XIX.

ELASTIC RELATIONS.*

WE call a riveting, simple, two-fold, or *m* fold, according as, in the direction of the force, there are one, two or *m* rivets or rows of rivets.

If a bar *S* is fastened by several rivets to an inelastic body *K* (Fig. 11), the rivet or rivet row *I* must take the whole stress, because that portion of *B* which we conceive to act upon *II* must first cause tension in *I II*, and therefore cause an elongation of *I II*. This, however, cannot occur, because *I*, by reason of the lack of elasticity of *K*, cannot give, and thus the whole stress comes upon *I*. Hence it follows, by union of an elastic body with an inelastic, compound riveting is useless; the rivet or rivet row next to the applied force must sustain the entire stress.

If the body *K* were elastic, but for the same or a greater cross-section less so than the bar *S*, then indeed a certain portion of *B* would act upon *II*, but only so much as corresponds to the elongation of *III*. The entire remaining force is sustained by *I*. For these reasons, compound riveting between bodies of different elasticity, such as steel and iron, cast and wrought iron, is not advantageous.

But similar relations may occur between bodies possessed

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* The elastic relations of riveting were first noticed by Schwedler, "Deutsche Bauzeit.," 1867, pp. 451, 463, 471. More recently the subject has been discussed by G. Müller, "Ztschr. d. östr. Ing. u. Arch. Vereins," 1874, p. 158.

of the same coefficient of elasticity. Thus, in Fig. 12, the diagonal represents the bar S, the flange-plate the body K, and this last evidently gives less than the first in the direction of the force B, so that at any rate rivet I will have more than the fourth part of B to sustain, and each of rivets II more than III. Where such unions cannot be avoided, we must be more lavish with rivets than otherwise necessary.

If two bodies, which for equal forces would elongate equally, it may be because for the same material they have equal cross-sections, are three or more fold riveted, they strive to elongate differently between two rivets, because the forces in the pieces either side are not equal. If, in Fig. 13, we indicate the stresses on the rivets by the distance apart, we have

$$I = I' \qquad II = II' \qquad III = III'$$
$$I\,II = I'\,II' = B - I \qquad II\,III = II'\,III' = B - I - II$$

The portions $I\,II$ and $II'\,III'$ are therefore acted upon by forces $B - I$ and $B - I - II$ of different intensities; the rivet I cannot give to the elongation of $I\,II$ demanded by the force upon that portion, and a portion of this force comes as compression upon I. An analogous relation holds for the portions $II\,III$, $I'\,II'$ and the rivet I'. The weak place of every more than two-fold riveting, lies at the outer rivets nearest the forces;* in such case, therefore, we should have rather *more* rivets than demanded by the ordinary calculation.

* This was first indicated in the "Ann. des Ponts et Chauss.," 1860, Vol. XX., No. 265. Before that it was customary to assume at once equal distribution of stress upon all the rivets. It would be very desirable to institute careful experiments upon the strength of compound riveted connections.

In many cases, we may, however, directly meet the difficulty. The elongation of the portions upon either side, and the most advantageous distribution of B upon the rivets, will be at least theoretically, when the cross-sections are as the forces which act upon them, or when in Fig. 13

$$\frac{F_1}{F_1'}=\frac{B-I}{B-I-II}=\frac{F_2'}{F}$$

This ratio, where a change of cross-section is allowable, may be attained by *per saltum* additions, but forms such as shown in Figs. 14, 31, 34 and 35 are to be preferred.

If two bars are so riveted that the stress acts in a line not coincident with one of the bars, there is exerted upon the other not only a force of extension or compression, but also a couple or bending force which causes an unequally distributed stress upon the bar and rivets. An arrangement of rivets as symmetrical as possible is in all cases to be recommended.

Schwedler * estimated for the connection shown in Fig. 15, upon the basis of the usual theory of flexure of Navier, the maximum stress per square centimetre in the outermost fibre, at

$$K=b\left(\frac{4}{\alpha}-\frac{3}{\alpha^2}\right)$$

where $b=\frac{B}{F}$ is the allowable stress of the compressed bar per sq. cent. and $\alpha=\frac{e'}{e}$. The maximum of K accordingly occurs for $\alpha=\frac{3}{2}$, for example for

$$\alpha=1 \quad \tfrac{5}{4} \quad \tfrac{3}{2} \quad 3 \quad 5$$
$$\frac{K}{b}=1 \quad 1{\cdot}28 \quad 1{\cdot}33 \quad 1 \quad 0{\cdot}68$$

* Schwedler, "Ueber Nietverbindungen," Dtsch. Bauzeit., 1867, p. 451.

that is, the resistance of the connection, if upon one side it extends beyond the double breadth of bar, is increased by removing the portion so exceeding. Theune has sought to prove this by experiments * with caoutchouc plates, from which it appeared that the weak place was generally not in the neutral fibre, but at o. The conclusion as to the inconsequence of the theory and the ground which Theune gives for the observed phenomena, cannot be regarded as correct. In his experiments the assumption of Navier's theory of flexure—deflections vanishingly small with reference to the length of piece—was not fulfilled at all, the force B could not be regarded as parallel to the axis of the splice. In Fig. 16, S denotes for caoutchouc the case of Schwedler, T that of Theune. In order to obtain the latter from the former, a new bending is necessary, by which the outermost fibre is to some extent relieved, and the weak place brought towards the cross-section o, which is now in a condition precisely similar to the cross-sections over the piers of a continuous girder. Nevertheless, these considerations, taken in connection with the preceding values for $\frac{K}{b}$, indicate that hardly a noticeable increase of stress upon the splice, in consequence of the projecting portion, is to be expected. The cutting off of this last will therefore in general give a better distribution of stress upon the rivets.

* Theune, "Ueber das Verhalten elastiger Platten bei unsymmetrischer Inanspruchnahme," Dtsch. Bauzeit., 1874, p. 76.

CHAPTER XX.

TOTAL CROSS-SECTION AND NUMBER OF RIVETS.

A RIVETED joint may be called a single shear, double shear, or i shear joint, according, as by the stress, one, two, or i changes of force direction occur (Figs. 17–22); for according as the force B shears the rivet in one, two, or i places, the whole, the half, or $\frac{1}{i}$th part of B acts to cause the shear of *each* section. Every i shear joint may be considered as decomposed into i single shear joints, as indicated in Figs. 19, 20 by the dotted line. The plate thickness of the single shear or "lap" joint we shall always denote by δ.

We have first to determine the total cross-section F_n of the rivets, afforded by any joint of bars or of plates. Let $max\ B$ be the greatest sliding force for this joint, then we have, denoting the allowable stress per square centimetre for shear by b', for tension and compression by b, for single shear joint,

$$F_n = \frac{max\ B}{b'} = \frac{max\ B}{\frac{4}{5} b} = \frac{5}{4} F \qquad (24)$$

for i shear joint,

$$F_n = \frac{max\ B}{i\ b'} = \frac{max\ B}{\frac{4}{5}\ i\ b} = \frac{5}{4i} F \qquad (25)$$

where F is the effective total cross-section of the bar upon which the stress in question, $max\ B$, is distributed.

The necessary number n of rivets may now be easily determined. If d is the diameter of rivet, or $\frac{1}{4}\pi d^2$ its area of cross-section, then

$$F_n = n \times \tfrac{1}{4}\pi d^2$$

and hence, with reference to (24) (25), for single shear joint,

$$n_1 = \frac{4}{\pi d^2}\frac{max\,B}{b'} = \frac{5}{\pi d^2}\frac{max\,B}{b} = \frac{5}{\pi d^2}F \qquad (26)$$

for i shear joint,

$$n_i = \frac{4}{i\pi d^2}\frac{max\,B}{b'} = \frac{5}{i\pi d^2}\frac{max\,B}{b} = \frac{5}{i\pi d^2}F = \frac{n_1}{i} \qquad (27)$$

The values of b and b' are not necessary here for the calculation if F is given.

That the double shear connection is double as strong as the single appears from the experiments of Fairbairn in Art. 18. The ratio is even still more favorable for the double connection, because in the single the cutting action of the edges, already noticed in Art. 18, is greater (Fig. 23). Straight shearing connections have also the advantage that there is no flexure of the plates and consequent springing of the rivet-heads (Fig. 23).

Often when it is difficult to dispose a sufficient number of single shearing rivets, we may advantageously make use of a fork-shaped joint as shown in Fig. 24. The rivets are then double shearing, and we need only half as many as for single shear. Still better is it when the fork-shaped connection can be made symmetrical.

The thicker the rivet, the fewer, of course, are necessary, and the farther apart they may be placed. We must, however, bear in mind that by the stress upon a single rivet the

corresponding compressive stress upon the surface of the hole increases; and finally a crushing of the edges of the rivet-hole may occur. If, indeed, in many cases a slight *burr* or upsetting is no disadvantage, still we must conclude, from the experiments of Gerber,* that the hole area per centimetre of projection must not be strained more than double the allowable tensile stress, or that

$$\frac{\pi d^2}{4} \cdot \tfrac{4}{5} b \leqq d\delta \cdot 2b$$

or,

$$d \leqq 3 \cdot 2\delta$$

This holds equally for single and i shearing connections, and in the latter case δ is the thickness of the single shear connections, into which we may consider the compound case divided.

REMARKS.

We have already pointed out that the stress of the rivets is made up of the transverse shear and the longitudinal strain due to cooling, without, indeed, being able to find a corresponding method of calculation. But, however, as long as the longitudinal strain is in full action, the friction thus produced of $R = 800$ to 1600 kil. per square centimetre of rivet cross-section, does not allow of a transverse shear at all; and if by vibrations or otherwise the friction has ceased to act, then there is no longer any longitudinal strain. In the intermediate conditions also no more unfavorable strain is to be anticipated, since for the shearing stress as ordinarily calculated the longitudinal strain has already considerably decreased at the moment when the shear commences.

* "Ztschr. d. Vereins dtsch. Ing.," 1865, p. 480.

The original longitudinal strain is not exactly determinable, since we do not know the coefficient of friction f. If we take this at $\frac{1}{6}$th and remember that R is due to friction upon two sides, we have for the longitudinal strain per square centimetre of rivet cross-section

$$L = \frac{R}{2f} = 1200 \text{ to } 2400 \text{ kil.}$$

This is large, even if we have to do with a permanent stress and the best fine-grained iron, and if the carrying strength has been raised by transgressions of the elastic limits (Art. 6). Since L in any case increases with the length of stem, it is not well, as already remarked, to have this exceed 10 c. m. (3·9 inches), and never 15 c. m. (5·9 inches). In such case it is best to use screw-bolts.

We can then, in accordance with all the results of theory and experiment down to the present time, put $F_n = \frac{5}{4}F$. If we depart from this, we should have a reason therefor. The following circumstances justify the choice of $F_n < \frac{5}{4}F$: (*a*) the fact that rivet-iron is in general better in quality than ordinary rolled iron; (*b*) the loss of strength which the piece undergoes by perforation, especially by punching (Art. 18). The following circumstances, on the other hand, are in favor of $F_n > \frac{5}{4}F$: (*a*) the unfavorable action of inclined strains in the rivet, if a greater stress is assumed; (*b*) the defective distribution of the stress upon the individual rivets in the case of more than double shear and by unsymmetrical grouping (Art. 19). These influences cannot be balanced once for all, and there is, therefore, no ground for departing from the general relation $F_n = \frac{5}{4}F$ in one or the other direction. We may in special cases, from theoretical grounds or constructive relations, be led to deviations (Art. 23), but the usual

reason that $F_n = F$ seems a simpler relation than $F_n = \frac{5}{4}F$, can hardly be regarded as satisfactory.

EXAMPLE.—Required to determine the number of rivets for the connection of the diagonals and verticals in the trussed frame of Fig. 6. Diameter of rivets $d = 2 \cdot 5$ cm. Rivets, single shear.

From (26) we have

$$n = \frac{5}{\pi d^2} F = 0 \cdot 25\, F$$

For the vertical *VI* we have found $F = 21 \cdot 1$, and therefore the necessary number of rivets

$$n = 0 \cdot 25 \times 21 \cdot 1 = 6$$

In similar manner, we find all the other values of n as shown in the following table:

Pieces:

	II,	*III*,	*IV*,	*V*,	*VI*,	*VII*,	*VIII*,	*IX*,	*X*,
$F =$	31·7	39·2	27·7	29·8	21·1	22·4	15·8	18·0	12·7
$n =$	8	10	7	8	6	6	4	5	4

As to the riveting of the apex plates with the flanges, see the example in Chap. 25.

CHAPTER XXI.

INDIRECT TRANSMISSION OF FORCE.

IN the preceding paragraphs it is assumed that the pieces riveted together lie directly in contact. Very often this is not the case, and then the relations are entirely different, a fact which has never been properly considered.*

Let it be required to unite the piece *I* with *III*, Fig. 25—that is, the stress *B* is to be transferred by rivets from *I* to *III*. Between *I* and *III* lies the arbitrarily strained piece *II*, which preliminarily we suppose not weaker than *I*. The force *B* can only pass directly from *I* to *II*, and for this transference we have at *A* the number of rivets n, calculated according to (26). If now, however, the piece *II* is not to be strained more than determined upon, it must before *A*, at *D*, be relieved by the same amount *B*, which also demands n rivets. The indirect transference of force by an intermediate plate requires then twice as many rivets as the direct.

That the force *K* can actually only pass direct from *I* to *II*, we may see from a glance at Fig. 26; for in order that the force may be transferred by a rivet to another piece, that rivet must be pressed against the rivet-hole in the direction of the force. We see thus from the figure

* Only in a single case for certain contact connections, considered by Schwedler. See Art. 27.

how II transmits to III, and that the rivets may be limited theoretically as shown by the dotted lines. If the piece were in compression, the rivet-holes would all bear against the opposite sides of the rivets.

In the case of two intermediate plates, the transference is as in Fig. 27. There are $3\,n_{,}$ rivets necessary and thus generally; *for every single shear connection the indirect force transference requires for* m *intermediate plates* $m + 1$ *as many rivets as for direct transference.* We have, therefore, from (26) for m intermediate plates,

$$n = (m + 1)\,n_{,} = \frac{5\,(m + 1)}{\pi\,d^2}\,\frac{max\ B}{b} = \frac{5\,(m + 1)}{\pi\,d^2}\,F \quad (28)$$

where F is the effective area of cross-section, whose stress is $max\ B$. In this equation formula (26) is included as a special case ($m = o$).

The above principle and formula (28) hold good also for every single shear connection, into which we may divide an i shear; and thus, for example, in Fig. 28, for the transference of the force B from I to III, $2 \times \frac{1}{2}\,n_{,} = 2\,n_{,}$ rivets are necessary. More than double shear connections do not occur in indirect force transference.

It is above assumed that the intermediate plates are not weaker than those whose stresses are transferred. This limitation is, however, not necessary, and our results hold generally good. If, for example, the intermediate plate in Fig. 29 were weaker than I, we may conceive the force transference as there indicated, if II is at least half as strong as I. If, then, the number of rivets for direct transference of a force, is denoted by the same index as the force itself, we have for the necessary number $n = 2\,n_{II} + 2\,(n_I - n_{II})$

$= 2\,n_I$ or just the same as above. In a similar manner, we may discuss any case of transference.

Filling pieces, such as are often unavoidable (Fig. 30), require only the single rivet number. They either move freely with the rivets without offering resistance, so that the transference from *I* to *II* takes place at *A*, or they are so situated that the force passes from the filling plate to the other at *D*. It is, however, to be observed, that in the first case the rivets are considerably influenced by flexure, which can be allowed for in the most unfavorable cases by the addition of a few rivets.

Applications in Chaps. 24–27.

CHAPTER XXII.

ARRANGEMENT OF THE RIVETS.

THE formulæ of the preceding paragraphs suffice in order to determine the number of rivets of given diameter for every connection; it remains only to make a few remarks as to their grouping. In this direction, we have already several valuable results in Art. 19. We have seen that the rivets, wherever possible, should be arranged symmetrically to the axis of the piece; that in the connection of bodies of equal or different elasticity, which yield differently in the direction of the force, simple riveting is the best; that in all cases a compound riveting causes a non-uniform distribution of stress disadvantageous to the outermost rivets nearest the forces, and that if there is no other objection, the rivets in the outer rows must be set as near as possible. Constructive relations and practical circumstances may often compel a disregard of these points; but the always disregarded friction affords in any case additional security, and by another method of design much of the force of these deviations may be neutralized.

In bar connections—for example, in the webbing of trussed frames—it is often essential to weaken the bar as little as possible by rivet-holes. If we should arrange the rivets of a diagonal as in Fig. 31, only *one* hole can be considered as weakening, and the effective cross-section is

$$F = b_1 \delta = (b - d) \delta$$

For if, indeed, in row *II* the breadth for application of the force is diminished by *d*, still this latter is also diminished by the force sustained by *I*. If, generally, the effective breadth of the entire bar is denoted by b_I, the following effective breadths hold for *II III* . . . . etc.;

$$b_{II} = \frac{B - I}{B} b_I = \frac{II + III + \ldots\ldots}{B} b_I$$

$$b_{III} = \frac{B - I - II}{B} b_I = \frac{III + IV + \ldots}{B} b_I$$

where *I*, *II*, *III* . . . . indicate the stresses sustained by the rivet rows *I*, *II*, *III*.

If of the breadths thus given, in any row there are Δ wanting, the effective area is $(b - d - \Delta)\,\delta$. We, therefore, never allow the rivet number in two successive rows to increase by more than the number in the outermost row. Then the necessary breadth exists so long as

$$d\,\delta\,b \leqq \frac{\pi d^2}{4} \cdot \tfrac{4}{5}\,b \text{ or } \delta \leqq \tfrac{5}{8}\,d$$

If $\delta > \frac{5}{8}d$, then in row *I* more rivets should stand than the increase in two successive rows; but such thick pieces are rarely used, and it must not be forgotten that δ is always the plate thickness for the *single* shear connections into which we can decompose any compound shear connection.

Since in Fig. 31, a less breadth is necessary for the transference of the force at *IV* than at *III*, and still there is less weakening by rivet-holes, the cross-section must be diminished in some other way. It has been shown in Art. 19 that such a diminution is desirable also in the interest of a

uniform distribution of the stress upon the rivet, and we may often therefore shape the ends as shown in Figs. 14, 31.

In pieces subjected to compression, we have not, properly speaking, to consider any weakening of the cross-section as caused by the rivets, since the stress is distributed over the entire breadth (Figs. 32, 33). It is also not so important here to put as few holes as possible in row *I*, but still it is well to consider at least a portion of the rivet-holes, as about one half, as causing a weakening of the cross-section, because a complete filling of the holes by the rivets is not to be expected in all cases.

In order to secure as uniform as possible a distribution of force upon the plate at the rivets as well as upon the rivets themselves, we may put where it is possible the rivets of each following row over the spaces between the preceding, generally over the middle of these spaces, so that only single "cleavage" can occur (Figs. 31, 40, 42).

If we consider the stress uniformly distributed upon the rivets, then each rivet has to sustain the stress upon a strip, whose breadth β is found from

$$\frac{\pi d^2}{4} \cdot \tfrac{4}{5}\, b = \beta \delta b$$

or

$$\beta = \frac{\pi}{5} \cdot \frac{d}{\delta} d$$

The stress upon every such strip must pass from it to the rivet; there must, therefore, in the grouping for any cross-section, be as many strips of the breadth β, side by side, as following rivets—assuming, of course, that not more rivets are used than the connection, according to 26, 27, theoretically demands. In symmetrical and direct transference, a strip

may run to its rivet in two halves of breadth $\frac{1}{2}\beta$. We thus arrive at a method of estimation of rivet grouping, first used by Schwedler (Figs. 34, 35, 39, 40), and it remains only to determine what breadth the strip should have for gradual transference *behind* the rivet. This is explained in what follows. The effective cross-section of the bar is naturally seldom an exact multiple of β, but in general somewhat less, because for each fractional number of rivets found by calculation the greatest whole number is taken, and often rivets are added.

The allowable minimum distance e of the rivets in the direction of the force, and the minimum distance r of the last row from the edge, are derivable from the condition that there must be equal safety both against shearing of the rivets and tearing through the dotted portions (Fig. 36). We consider the shearing areas of these portions as included only between the tangents to the contiguous hole edges, on account of the diminution of resistance already referred to in Art. 18, in the immediate vicinity of the hole, and where also in punched holes small cracks readily occur. We have, therefore,

$$P = \frac{\pi d^2}{4} \cdot \tfrac{4}{5} b = 2(e - d)\ \delta \tfrac{4}{5} b$$

whence

$$e = \left(1 + \frac{\pi}{8}\frac{d}{\delta}\right) d \qquad (30)$$

From practical grounds—as, for example, on account of the necessary space for the striking of the riveting hammer—we are almost always obliged to exceed this distance.

The least distance of the outermost rivet row from the edge is less than e by $\frac{1}{2}d$ (Fig. 36), hence

$$r = \left(1 + \frac{\pi}{4}\frac{d}{\delta}\right)\frac{d}{2} \qquad (31)$$

Equations (30) (31) also show that behind each rivet a strip is necessary of at least $\frac{\pi}{8}\frac{d}{\delta}d$, with reference to the stress upon *this* rivet, so that the dimensions of the strip surrounding a rivet, as in Fig. 37, are given. In *many-shear* or compound-shear connections, we must put in all the formulæ, for δ the thickness of plate for the single-shear connections into which the first may be divided (Art. 20).

EXAMPLE.—In practical cases, where we are free to assume δ at pleasure, $\delta = \frac{1}{2}d$ is a very suitable value. Required to determine the quantities β, e, r, under this assumption, for the customary rivet diameters.

We have from (29), (30), (31),

$$\beta = 1{\cdot}26d \qquad e = 17{\cdot}9d \qquad r = 12{\cdot}9d$$

Hence in millimetres for

$d =$ 20	21	22	23	24	25	26	27	28	29	30
$\beta =$ 25·2	26·5	27·7	29·0	30·2	31·5	32·8	34·0	35·3	36·5	37·8
$e =$ 36	38	39	41	43	45	47	48	50	52	54
$r =$ 26	27	28	30	31	32	34	35	36	37	39

These values of e and r must generally be in practice somewhat increased.

CHAPTER XXIII.

RIVETING OF PLATES.

IN the riveting of plates, the grouping of the rivets is beforehand determined, and the rivets must be arranged in one or two rows, uniformly spaced. With two plates of the same material and of equal strength, double riveting is in general preferable to single; the necessary number of rivets remains the same, the stress is still distributed uniformly upon all the rivets (Art. 19), but the effective cross-section F is greater, and the strength in the rivet-seam is affected less by the holes. As to the *tightest* joint, double riveting may not, in some cases, be so good.

The necessary number of rivets in the present case is not generally found from the formulæ of Art. 20; it is given at once by the rivet distribution, and this follows from the condition that the resistance of the rivet to shearing and of the plate to tearing shall be the same. We have therefore for single and double riveting (Figs. 38–40), since here there is but single shear, respectively

$$\frac{\pi d^2}{4} \cdot \tfrac{4}{5} b = (D-d)\,\delta b$$

$$2\,\frac{\pi d^2}{4} \cdot \tfrac{4}{5} b = (D-d)\,\delta b$$

whence

$$\text{for single riveting} \quad D = \left(1 + \frac{\pi}{5}\frac{d}{\delta}\right) d \qquad (32)$$

$$\text{for double riveting} \quad D = \left(1 + \frac{2\pi}{5}\frac{d}{\delta}\right) d \qquad (33)$$

More than double riveting would be, by reason of the lack of uniformity in the distribution of the stress, not advantageous.

From $\alpha = \frac{D-d}{D}$, we have the ratio of the effective to the total cross-section, and thus for

$$\text{single riveting} \quad \alpha = \frac{1}{1 + \frac{5}{\pi}\frac{\delta}{d}} \qquad (34)$$

$$\text{double riveting} \quad \alpha = \frac{1}{1 + \frac{5}{2\pi}\frac{\delta}{d}} \qquad (35)$$

and we have for the effective section, if F' is the total cross-section,

$$F = \alpha F' \qquad (36)$$

We see that, so soon as F and the ratio $\frac{d}{\delta}$ are known, the necessary total cross-section F', and therefore the necessary plate thickness δ, may be found.

For example, from (34) and (5), we obtain

for $\frac{d}{\delta} =$	1·5	2	2·5	3
for single riveting $\alpha =$	0·49	0·56	0·61	0·65
for double riveting $\alpha =$	0·65	0·72	0·76	0·79

If the stress upon the plate acts perpendicular to the direction of rolling or fibre, and if, therefore, we take the stress only $\frac{9}{10}$ths as great as when longitudinal, or take only $\frac{9}{10}b$ (Art. 5), we have just as above, for

$$\text{single riveting}\quad D = \left(1 + \frac{2\pi}{9}\frac{d}{\delta}\right)d$$

$$\alpha = \frac{1}{1 + \dfrac{9}{2\pi}\dfrac{\delta}{d}}$$

$$\text{double riveting}\quad D = \left(1 + \frac{4\pi}{9}\frac{d}{\delta}\right)d$$

$$\alpha = \frac{1}{1 + \dfrac{9}{4\pi}\dfrac{\delta}{d}}$$

Thus, for example, if

	$\frac{d}{\delta} = 1\cdot5$	2	$2\cdot5$	3
for single riveting	$= \alpha\, 0\cdot51$	$0\cdot58$	$0\cdot64$	$0\cdot68$
for double riveting	$\alpha = 0\cdot68$	$0\cdot74$	$0\cdot78$	$0\cdot81$

The necessary effective cross-section is here in the ratio of 10 : 9 greater than above, but a part of this may be lessened by the somewhat less weakening.

The formulæ would also hold good for *many-shear* connections, if these occurred with plates, and δ then must be taken as so often indicated for this case.

In the riveting of plates it is permissible to take the total cross-section of the rivets $F_n = F$, instead of, as formerly, $F_n = \frac{5}{4} F$, for in plates the shearing strength in the practically

important directions is often found equal to and even greater than the tensile strength in the direction of fibre. Also the reasons advanced in Art. 20 in favor of $F_s < \frac{4}{5} F$ hold good, since we have here a uniform distribution of stress upon the rivets. For plates, therefore, a dissipation of the resistance of friction is less to be feared, in many cases because the joints without it would no longer be tight and the riveting would be useless. If, then, we take $F = F_s$ (that is, the shearing strength equal to the tensile strength), we have in the same way as above,

$$\text{for single riveting} \quad D = \left(1 + \frac{\pi}{4}\frac{d}{\delta}\right)d \qquad (32a)$$

$$\text{for double riveting} \quad D = \left(1 + \frac{\pi}{2}\frac{d}{\delta}\right)d \qquad (33a)$$

and for the ratio of the effective to the total cross-section,

$$\text{for single riveting} \quad \alpha = \frac{1}{1 + \frac{4}{\pi}\frac{\delta}{d}} \qquad (34a)$$

$$\text{for double riveting} \quad \alpha = \frac{1}{1 + \frac{2}{\pi}\frac{\delta}{d}} \qquad (35a)$$

Therefore, if, for example,

	$\frac{d}{\delta} = 1\cdot5$	2	2·5	3
for single riveting	$\alpha = 0\cdot54$	0·61	0·66	0·7
for double riveting	$\alpha = 0\cdot70$	0·76	0·80	0·82

For the case where the stress upon the plate is perpen-

dicular to the direction of fibre, and the tensile strength is $\frac{9}{10}$ths of that in the direction of fibres, we have for

$$\text{single riveting} \qquad D = \left(1 + \frac{5\pi}{18}\frac{d}{\delta}\right) d$$

$$\alpha = \frac{1}{1 + \frac{18}{5\pi}\frac{\delta}{d}}$$

$$\text{double riveting} \qquad D = \left(1 + \frac{5\pi}{9}\frac{d}{\delta}\right) d$$

$$\alpha = \frac{1}{1 + \frac{9}{5\pi}\frac{\delta}{8}}$$

We have, therefore, for

	$\frac{d}{\delta} = 1 \cdot 5$	2	2·5	3
for single riveting	$\alpha = 0 \cdot 57$	0·64	0·69	0·72
for double riveting	$\alpha = 0 \cdot 72$	0·78	0·81	0·84

Formulæ (32 *a*) to (35 *a*) are given by Grashof and others.

CHAPTER XXIV.

DISTRIBUTION OF RIVETS IN PLATE-GIRDERS.

THE riveting of the plate-girder demands special consideration. The distance apart of the rivets in each row or the rivet distribution must here be determined.

The union of the flange with the vertical plate is exclusively effected by row I, Fig. 41. Without this the vertical plate could move freely in the space between the two angle irons. The rivet row I must resist the greatest force acting to produce such motion. Let now the horizontal shear at I per unit of length be H_I and the allowable stress of a rivet be N, then we have per unit of length,

$$n = \frac{max\ H_I}{N}$$

rivets. If these rivets are to stand as here in a single line, then the distance from rivet to rivet must be

$$e_I = \frac{1}{n} = \frac{N}{max\ H_I} \qquad (37)$$

The rivets in row I are in double shear, and therefore

$$N = 2\,\frac{\pi d^2}{4} \cdot \tfrac{4}{5}\, b$$

This value is, however, only allowable when the pressure upon the hole area is not too great, which is always the case (Art. 20) when $d > 3 \cdot 2\ \delta$. Since in the present case $\delta = \frac{1}{2}\,\Delta$ (where Δ is the thickness of the vertical plate, Chap-

ter XX.), we have always $d > 3 \cdot 2\, \delta$; and if, therefore, the pressure upon the area of the rivet-holes is not to be too great, we must only allow

$$N = d\, \varDelta \cdot 2b$$

so that, according to (37)

$$e_I = 2\, d\, \varDelta \frac{b}{max\ H_1}$$

In this equation we can now insert the exact value of $max\ H_1$. As, however, a hair-splitting accuracy is superfluous, we may conclude as follows:

The horizontal shear per unit of length is greatest in the neutral layer; its value is

$$H_o = \frac{V_x}{v}$$

where V_x is the total vertical shear in the cross-section x and v is the distance between the centres of compression and tension. From the neutral layer to I the horizontal shear, as is well known, diminishes but little, and this diminution we can best regard by taking simply instead of the smaller value v the greater value v_o between the centres of the flanges, so that approximately at I we have

$$H_I = \frac{V_x}{v_o}$$

The decrease of the horizontal shear is less, the thinner the vertical plate and the larger the flange, and when the vertical plate vanishes the equation is exact, because then v and v_o are identical. We have then for the number of centimetres between the rivets in row I,

$$e_I = 2\, d\, \varDelta\, v_o \frac{b}{max\ _xV}$$

where b is the allowable tensile stress per square centimetre, $max\ V_x$ the greatest occurring vertical shear at x without reference to its sign, d the diameter of rivet, Δ the thickness of the vertical plate, v_0 the distance between the centres of the flanges.

Formula (38) holds good as well for a constant as for a variable b. Assuming the latter, we have according as V_x has at x always the same or alternating sign, for wrought-iron, as in Art. 17,

$$b = 700\left(1 + \tfrac{1}{2}\frac{min\ V_x}{max\ V_x}\right)$$

$$b = 700\left(1 - \tfrac{1}{2}\frac{max\ V'_x}{max\ V_x}\right)$$

In the last formula, $max\ V_x$ is the greatest vertical shear generally, which acts at x, $max\ V'_x$ is the greatest of the two opposite stresses, but both are to be introduced irrespective of the signs denoting their direction. If now e_I also varies according to (38) with $\dfrac{b}{max\ V_x}$, still in all practical cases this quotient and therefore e_I will be least at a support, as well for the simple as for the continuous girder. If, then, we wish to make e_I constant, we need only find its value at the supports; and since at the supports V_x has always the same sign the first formula applies. However, e_I does not vary as much as by the old method of calculation, because not only V_x decreases towards the middle, but also b. A constant e_I is to be recommended for small girders. For longer ones, we may take e_I somewhat greater towards the centre of the span, a practice for which the experienced constructor needs no calculation.

If in the calculation of a simple plate-girder we suppose a uniformly distributed load, we have at the supports, if p is the dead load and q the total load per unit of length,

$$max\ V_x = \tfrac{1}{2} q\, l \qquad min\ V_x = \tfrac{1}{2} p\, l$$

or

$$b = 700 \left(1 + \tfrac{1}{2} \frac{p}{q} \right)$$

and hence, by substitution, we have for the least distance between the rivets (that is, for the greatest allowable distance at the supports)

$$e_I = 2800\, \Delta\, v_0 \frac{1 + \frac{p}{2q}}{q\, l} \qquad (38a)$$

where Δ and v_0 are to be measured in centimetres.

From (38) and (39), we see that the rivet distribution is proportional to the thickness of the vertical plate or web, and to the distance between the centres of the flanges, so that we are able, if desired, to increase the distance between the rivets.

For the rivet row at II (Fig. 41) we have to consider how many horizontal plates there are. For more than one we have indirect transference of force, since the stress must always be transferred to the last applied plate. Hence the necessary number of rivets per unit of length is, if $max\ H_{II}$ is the greatest horizontal shear at II and μ the number of applied plates at the cross-section x in question,

$$n = \mu \frac{max\ H_{II}}{N}$$

and the greatest allowable space between rivets for r rows is

$$e_{II} = \frac{r}{n} = \frac{r\, N}{max\ H_{II} \mu}$$

Generally $r = 2$, and since the rivets in II are single shear and the condition $d \leqq 3 \cdot 2\,\delta$ is always fulfilled, where δ is the thickness of the horizontal angle-iron leg, we have

$$N = \frac{\pi d^2}{4} \cdot \tfrac{4}{5} b$$

and therefore

$$e_{II} = \frac{2\,\pi d^2}{5\,\mu} \cdot \frac{b}{max\, H_{II}}$$

Now, the horizontal shearing force at II is to that at I approximately as the cross-section of the horizontal plates to the cross-section of the entire flange—that is, indicating this ratio by γ

$$H_{II} = \gamma\, H_{I} = \gamma \frac{V_x}{v_0}$$

Hence, for the rivet distribution at II, we have

$$e_{II} = \pi\, d^2\, v_0 \cdot \frac{2}{5\,\mu\,\gamma} \cdot \frac{b}{max\, V_x} \qquad (40)$$

From (38) and (40) the ratio of the theoretical rivet-distribution at II and I is for any point x

$$e_{II} = \frac{\pi\, d}{5\,\mu\,\gamma\,\Delta}\, e_I$$

where μ is the number of the horizontal plates, γ the ratio of their total cross-section to that of the flange, Δ the thickness of the vertical plate, and d the diameter of rivet.

Assuming that at any point $\mu = 3$, $\gamma = \frac{3}{5}$, $d = 3$ cm., $\Delta = 1$ cm., we have from (41) $e_{II} = 1 \cdot 05\, e_I$; we see, therefore, that for ordinary proportions for one and two horizonal plates, the distance between rivets in II is greater, for three horizontal plates the same, and for more plates less, than in I at the

same place. If, therefore, as often happens, we wish the same rivet distance at I and II (Fig. 42), we may, with three horizontal plates, make the same disposition as at I, without specially calculating e_{II}.

EXAMPLE.—In two simple plate-girders of $l = 7$ and 10 metres span, and $v_0 = 75$ and 110 cm. distance between the centres of the flanges, let $p = 900$, $q = 8100$ kil., and $p = 1000$, $q = 7000$ kil. respectively. In both cases, the thickness of the vertical plate $\Delta = 1$ cm., and rivet diameter $d = 2 \cdot 5$ cm. To find the least rivet distance e_I.

From (38a) we have at once

$$\text{for } l = 7 \quad e_I = 2800 \times 2 \cdot 5 \times 75 \, \frac{1 + \frac{1}{18}}{8100 \times 7} = 10 \text{ cm.}$$

$$\text{for } l = 10 \quad e_I = 2800 \times 2 \cdot 5 \times 110 \, \frac{1 + \frac{1}{14}}{7000 \times 10} = 12 \text{ cm.}$$

If the load is not uniformly distributed, formula (38) must be applied.

REMARK.—We see that by the present method of calculation very plausible distances are given, while we are accustomed to hear that calculation gives values practically much too great. If, indeed, in the deduction of the formulæ, we had had no regard to the allowable pressure upon the hole area, we should have found the distance e_I greater, in the ratio

$$\left(2 \frac{\pi d^2}{4} \cdot \tfrac{4}{5} b\right) : (2 \, d \, \Delta \, b) = 0 \cdot 63 \, \frac{d}{\Delta}$$

But then the pressure upon the hole area would have been $0 \cdot 63 \, \frac{d}{\Delta}$ times as great, and thus for $\Delta = 1$ and $d = 3$, almost as great again as allowable.

CHAPTER XXV.

FLANGE-RIVETING IN FRAMED TRUSSES.

FOR framed trusses—that is, for systems whose separate members sustain only an axial tension or compression—a definite spacing of the rivets for the flanges cannot be theoretically determined. The formulæ here and there deduced for this case rest upon erroneous assumptions. Upon the connection of the webbing with the flange-plate enough has already been said in Arts. 20, 21.

If the *compressed* flange between two apices consists of several bars, rivets are necessary in order so to bind these bars together that they may act as a whole against the stress. For the resistance of a piece to flexure is proportional to the least moment of inertia with reference to the axis through the centre of gravity of the cross-section; and, therefore, the resistance of the flange, *as a whole*, is far greater than the sum of the resistances of the component parts.

For the same reasons, the compressed flange should always receive a cross-section possessing the greatest possible moment of inertia, or an extended form, while for flanges in tension where only uniform distribution of stress is of importance, we may have, in general, more compact forms.

A riveted connection of the individual parts of a compressed flange, is, for a proper arrangement of the apices,

not necessary theoretically. "Jump joints" or pieces which simply abut, are nevertheless always riveted in order to guard against atmospheric influences; moisture remains longest and rust occurs easiest in narrow cracks. Often, however, the flange is composed of several sufficiently distant and individually compacted portions.

As rivet spacing for the flanges of framework, we may take for rivets from 2 to 3 cm. thick (0·8 to 1·2 inches) a distance apart of about 14 to 20 cm. (5·5 to 8 inches). For extended flanges, we can set them even further apart, especially when the pieces to be connected are weak, and therefore no very great force is necessary for the production of a tight joint. We must always have care that the cross-section of the flange be weakened by as few rivet-holes as possible.

If we suppose the web-pieces of a framed girder riveted to an apex plate, but separated from the flanges, then this plate corresponds exactly to the vertical plate in plate-girders. The rivets by which the apex plate now is united with the flange have the same office to perform as the rivet row *I* for the plate-girder (Fig. 41), and the rivets by which the stress is transferred from the directly adjacent to the other portions of the flange, correspond to the rivet row *II*. A transference of any forces outside of the apex points—that is, outside of the space enclosed by the apex plate—does not occur in framed trusses. In plate-girders, the transference takes place at all cross-sections; the apex points are infinitely close to each other.

We must therefore take pains that the axis of symmetry of the web-members intersect the axis of symmetry of the flange (Fig. 46), because every eccentric longitudinal stress

may cause flexure, and hence an unequal distribution of force.* This long-recognized fact has been lately controverted, by assuming that for ordinary, just the same as for plate-girders, the outermost fibres of the compressed flange as well as of the tension flange are more strained than those nearer the interior, an assumption not justified by theory. From the same point of view, formulæ have been deduced for the rivet spacing of the flanges, which naturally have no value. When the forces act eccentric, the flanges are in the condition similar to those of the continuous girder, which also is partially fastened over its supports.

The number of rivets by which the apex plate is united with the flanges, must be great enough in order to transfer the resultant R of the stress in the web-members to those portions of the flange designed to sustain the stress R. In most cases, then, we have an indirect force transfer (Art. 21), and the necessary number of rivets is, when m is the number of bars which intervene between the apex plate and that piece in which R is to take effect, according to (28)

$$n = (m + 1)n_{,} = \frac{5\,(m + 1)}{\pi\, d^2}\,\frac{max\, R}{b} \qquad (42)$$

of which at least $\frac{1}{m}$ must meet the last-named piece.

In the cross-section shown in Fig. 43, for example, the strength of the angle iron in general varies to correspond with the increasing flange force; R must therefore pass over

* This may occur in similar manner as pointed out by Schwedler for the splice-plates, and so far as regards the fact of flexure, confirmed by the experiments of Theune. See also Laissle und Schübler, IV. Ed. 1874, p. 107. The general theory of eccentric and transverse loaded pieces is given in an essay by the author in "Ztschr. f. Math. und Phys.," 1874, p. 536.

into the angle iron, and $m = 1$, because there is but one intervening plate. In the cross-section shown in Fig. 44, we also have $m = 1$, for although R is usually transferred to the horizontal plate, yet R is transferred by those rivets which meet the apex plate, only to the upper angle iron; for the further transference to the horizontal plates, we have the rivets in the horizontal legs of the angle irons, whose number generally for $m + 1$ horizontal plates is also found by (42). In both cases, therefore, we have for the connection of the apex plate

$$n = \frac{10}{\pi d^2} \frac{max\, R}{b} \qquad (43)$$

This special formula finds particular application for principal trusses, and for such can be always used without further discussion, if there is no point made of a few rivets too many. The number n, however, does not include the rivets necessary for the connection of the web-pieces with the apex plate, even when these meet the flange.

If the resultant is transferred partly into directly contiguous plates, $\frac{1}{v}$ into a piece separated from the apex plate by one plate, $\frac{1}{w}$ into one separated by two plates, we have

$$n = n_1 + \frac{n_1}{v} + 2\frac{n_1}{w} = \frac{5}{\pi d_2} \frac{max\, R}{b} \left(1 + \frac{1}{v} + \frac{2}{w}\right) \qquad (44)$$

Here, in the most unfavorable case, the value within the parenthesis will reach nearly 2 (usually $\frac{2}{w} = 0$, because there is no transference with 2 intermediate plates), so that formula (43) can never give too few rivets.

If α is the angle included by two web-members simulta-

neously acting and meeting at the same apex, we have from Fig. 45,

$$R = \sqrt{X^2 + Y^2 - 2XY\cos\alpha}$$

where X and Y are the two simultaneous stresses without reference to sign. We must now determine $max\ R$, and for this none of the formulæ deduced for various forms of truss and apex points hold good. On the other hand, we know from the statical calculation $max\ X$ and $max\ Y$, which generally do not occur indeed, at the same moment. We may now supersede all special rules and obtain $max\ R$ in many cases somewhat too great, in others exactly, and in none too small, by simply putting

$$max\ R = \sqrt{max\ X^2 + max\ Y^2 - 2\ max\ X\ max\ Y\cos\alpha} \quad (45)$$

Of course, a more exact or more convenient determination of $max\ R$ is not excluded (see example below), but it is certainly better to compute a little than not at all, and $max\ R$ and n *may* indeed be taken somewhat too great, since a uniform distribution of stress upon the rivets in the apex-plate is only rarely to be expected.

In the formulæ, for n we should properly insert for b the allowable tensile stress per square centimetre for the strain ratios $\frac{min\ R}{max\ R}, \frac{max\ R'}{max\ R}$; but it is more convenient to take for b the smallest of those two values, which, in the calculation of the dimensions of the two web-members, have been already given by $max\ X$ and $max\ Y$. This value is in certain cases exact, in others somewhat too small, so that again n is found rather too great.

EXAMPLE.—To find the necessary number of rivets for

the connection of the apex-plate in the framed girder given in Art. 14 (Fig. 6). Form of flanges as in Figs. 43, 44, diameter of rivets $d = 2 \cdot 5$ cm. (1 inch).

We have from (43)

$$n = 0 \cdot 51 \frac{max\ R}{b}$$

and since here $\alpha = 45°$, we have

$$max\ R = \sqrt{max\ X^2 + max\ Y^2 - 1 \cdot 4\ max\ X\ max\ Y}$$

For the apex *IV V*, for example, we have

$$max\ R = \sqrt{21000^2 + 22100^2 - 1 \cdot 4 \times 21000 \times 22100} = 16700 \text{ kil.}$$

In the calculation of *IV*, b was 758, and for *V*, $b = 742$ (Example in Art. 14); therefore, according to the above,

$$n = 0 \cdot 51 \frac{16700}{742} = 12$$

In the same way, the rivet number for the other apex-points of the upper flange may be calculated.

In the statical calculation of the above framework only the upper apices were supposed loaded. In order now to find *max R* for the lower apices more conveniently than by formula (45), we must remember that both diagonal and verticals, when they meet at an unloaded apex in a girder with parallel flanges, undergo equal vertical strains, and that both, and therefore also R, reach simultaneously their maxima and minima. We have, therefore, for the apices of the lower flange (Fig. 46),

$$max\ R = max\ X \cos \alpha$$

or since $\alpha = 45°$,

$$max\ R = max\ X$$

so that the exact value of *max* R, and also the exact value of b are now known.

For the apex $IX\ X$, for example, *max* $R = 6750$, and b is as in the calculation of IX and X, 531. Hence

$$n = 0 \cdot 51 \frac{6750}{531} = 7$$

In the same way n is found for the other apices of the lower flange. The results for upper and lower flanges are then as follows:

Apex	*II III*,	*III IV*,	*IV V*,	*V VI*,	*VI VII*,	*VII VIII*,	*VIII IX*,	*IX X*
max $R =$	21200	21000	16700	15625	11900	10875	7880	6750
$b =$	758	758	742	742	688	688	531	531
$n =$	14	14	12	11	9	8	8	7

In these numbers the necessary rivets for the connection of the web-members (see Art. 20, Example) are not included, even when these, as is usually the case for the verticals, meet the flange. On the other hand, n holds for the entire force transmitted to the apex. If, therefore, the apex-plates are used as in Figs. 43, 44, for each apex, each of them requires $\frac{1}{2}\,n$ rivets.

CHAPTER XXVI.

RIVET-SPACING FOR LATTICE GIRDERS WITH STAY-PLATES.

THE data of the preceding paragraphs hold good evi dently as well for simple as for compound framework with apex-plates. The application of continuous stay-plates instead of apex-plates is in general not advantageous; the rivets are not uniformly strained (Art. 19) and the stay-plates at the connection with web-members are *over-strained*—the last because at these places the stay-plate has to sustain the flange-stress, and also the stress of the web-members. If, however, the web-members are close together, it is often convenient to replace all apex-plates by a continuous stay-plate. We should, in such case, having found the most unfavorable stress of the stay-plate from the calculation for apex-plate, at least not include the whole of it in the flange, and should also use rather more rivets for the web-members than otherwise.

The lattice-bars united with the stay-plates form a web corresponding perfectly to the vertical plate of plate-girders. The transference by the stay-plates to the flanges of the forces received by them from the lattice-bars takes place no longer at certain apices, but in a constant manner, and hence, in opposition to the trussed girder with apex-plates, a definite number of rivets is capable of being determined.

The spacing of the rivet row *I* (Fig. 47) is, if the flanges

are parallel, entirely similar to the plate-truss, as given in (37), only that we must have $N = 2\frac{\pi d^2}{4} \cdot \frac{4}{5} b$. We thus obtain

$$e_I = \frac{2}{5} \pi d^2 v_0 \frac{b}{max\ V_x}$$

Here, we have always (for continuous girder also), $\frac{b}{max\ V_x}$, least at a support, and therefore the least rivet space occurs there, and because V_x has always the same sign, we have for the least rivet space

$$b = 700 \left(1 + \frac{1}{2} \frac{min\ V_x}{max\ V_x}\right)$$

Towards the middle of the truss or span, V_x may have different signs, so that if the rivet-spacing is to vary we may also apply

$$b = 700 \left(1 - \frac{1}{2} \frac{max\ V'_x}{max\ V_x}\right)$$

For the special case of the simple lattice girder, when for the statical calculation a uniformly distributed dead load p and a total load q is assumed, we have for the least rivet distance

$$max\ V_x = \frac{q\,l}{2} \quad b = 700 \left(1 + \frac{1}{2} \frac{p}{q}\right) \quad \text{and}$$

$$e_{II} = 560\ \pi d^2 v_0 \frac{1 + \frac{p}{2q}}{q\,l} \qquad (46a)$$

where d and v_0 are in centimetres, as is also e_{II}

For the rivet-spacing at I, we have, when, as in Fig. 47,

two rows are used, precisely as in Art. 24, and for similar signification of all the quantities

$$e_{II} = \pi\, d^2\, v_0 \cdot \frac{2}{5\mu\gamma} \cdot \frac{b}{max\ V_x} \qquad (47)$$

The relation between e_{II} and e_I at any point is accordingly

$$e_{II} = \frac{e_I}{\mu\gamma} \qquad (48)$$

where e_I is the theoretical rivet-spacing at I, μ the number of horizontal plates, and γ the ratio of their combined cross-section to the entire cross-section of the flange, all taken at the same given point.

Equations (46) (47) apply to the special case of horizontal flanges. If at any point the flange is inclined to the horizon by an angle α, the transferred force will be $\frac{1}{\cos\alpha}$ times greater, and the corresponding rivet-spacing therefore $\frac{\cos\alpha}{1}$ times less than for the same v_0 for horizontal flanges. We have, then, for the rivet-spacing at this point

$$e_I = \tfrac{2}{5}\, \pi\, d^2\, v_0 \cos\alpha \frac{b}{max\ V_x} \qquad (49)$$

v_0 being the vertical distance between the flange centres.

If the flange has a form, by which at I there are two rows of single instead of one row of double shear rivets, the formulæ for e_I still hold; if two rows of double shear, or four rows of single shear occur, we may take e_I twice as great. In similar manner, we may conclude as to the proper spacing in (II), without going back to the general formulæ (37), (39).

EXAMPLE.—For bridges of the spans given below, loaded

with the heaviest locomotives (those of the "Hessischen Ludwigsbahn"), taking with Schwedler $p = 800 + 30\,l$, we may, according to Schäffer, calculate q.* It is required to determine for a parallel flanged lattice girder the least rivet-spacing e_I, under the assumption of a single row of double shear rivets (Fig. 47), or two rows of single shear rivets, v_o being $\frac{1}{10}\,l$, and d = 2·5 cm. (1 inch).

From (46 a) we have

$$e_I = 10996\, v_o \frac{1 + \frac{p}{2q}}{q\,l}$$

Where v_o is to be inserted in centimetres; thus, for example, for $l = 20$ m.,

$$e_I = 10996 \times 200 \frac{1 + \frac{1}{10}}{7000 \times 20} + 17\cdot0 \text{ cm.}$$

In similar manner, the other values are computed:

$l =$	8	10	15	20	30	40	50 metres.
$p =$	1040	1100	1250	1400	1700	2000	2300 kil.
$q =$	9440	8400	7050	7000	6900	7100	7200 kil.
$e' =$	12·3	13·9	16·9	17·3	17·9	17·7	17·7 centimetres.

We have purposely taken v_o only $\frac{1}{10}\,l$ in order that the least rivet-spaces in practical cases may seldom be less than here computed. For such large values, it may often be advantageous to make e_I constant for the whole length of girder.

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* Compare Erbkams, "Ztschr. f. Bauwesen," 1874, p. 407.

CHAPTER XXVII.

CONTACT CONNECTIONS

As regards the arrangement of riveting at contact joints, we may distinguish as to whether the piece is extended or compressed. For while, in the first case, the connections alone receive the stress, in the second, we may consider the force as transferred directly (Figs. 48, 49). The last, however, will only occur when the joints are in perfect contact, which can neither be assumed as invariable nor invariably effected. Since also the joints are subjected to side shocks, it is customary in practice to proportion the contact connections for compressed portions precisely as for extended, and only where the requirements of construction render a diminution of the splice length or rivet number desirable, to regard the allowability of such diminution.

Double splices, or "fish-plates," are to be preferred to single. If extended pieces are united by single fish-plates, we have a couple producing flexure, which may even spring the rivet-heads (Fig. 50). Single fish-plates occasion in compressed pieces even a greater danger by buckling. These points are, however, only of importance for single riveting, and in many cases—as, for example, for flanges, where single splice-plates are necessary, because the other side of the compressed piece is not free—the adjacent piece prevents sufficiently all flexure or buckling. In the last case, it is much more objectionable, that in all probability a portion of the

stress does not affect the connections at all, but passes into the adjacent piece; and such pieces, according to experiments made during the construction of the Britannia bridge under certain circumstances, were found to sustain $\frac{3}{2}$ of the stress properly assigned to them.

Since the splice-plates must sustain the same stress as the touching pieces, the effective cross-section of the first must equal the effective cross-section of the last, and hence double splice-plates need only be half as thick as single. Double plates also require in general only half as many rivets, because these are then in double shear. With the necessary number of rivets, the necessary length of splice also diminishes, so that double splices afford a saving of material. It is also in their favor that the stress is more uniformly distributed upon the rivets, the fewer there are following each other in the direction of the force.

We should, therefore, whenever possible, make use of double splice-plates.

The stress upon the pieces in contact is transmitted by the rivets to the splice-plates. Since therefore the rivets sustain upon each side a shear, corresponding to the tension, compression, or shear of the piece, the necessary number of rivets upon each side of the joint is from Art. 20 (Figs. 48, 51)

$$\text{for single splice} \quad n_1 = \frac{5}{\pi d^2} F \qquad (26)$$

$$\text{for double splice} \quad n_2 = \frac{5}{2 \pi d^2} F = \frac{1}{2} n_1 \qquad (27)$$

where F is the effective cross-section of the piece subjected to impact.

These formulæ hold good only for the case, however,

that the splices are in direct contact with the piece. If an intervening plate has to be inserted between the two, the splice should have twice as many rivets, and it must accordingly be about twice as long.* We have here again a case of indirect force-transference, and the remarks of Art. 21 hold good. In order, however, to make the case particularly clear in this instance, it may be observed (Fig. 52) that, first, the plate *II* must receive the stress upon *I*, and that the portion *aa* acts in fact as a splice for *I*. In order, however, that this may take place without overstraining, the piece *II* must already have been previously strained by the amount of the stress in *I*, for which once more n_1 rivets are necessary upon either side. The further discussion of the case is to be found in Art. 21.

If the preceding has been thoroughly comprehended, there will be no difficulty in estimating any more complicated connection. Thus, very often, but indeed seldom with sufficient reason, two plates have joints at the same place. If the joints coincide (Fig. 53), we have simply two splice-plates of the same thickness as the plates, and upon each side of the joints n_1 rivets, or a total of $2n_1$ rivets. The joints, however, do not usually coincide (Fig. 54), because this would make a weak place. In such case, the splice-plates need only be half the thickness of the plates, and $2n_1 + 2n_2 = 3n_1$ rivets are necessary, as shown by the representation of the force transference in Fig. 54, where full lines indicate the whole and dotted lines the half of the force P acting in any piece. It is not necessary to lengthen the splices in Figs. 54, 55, as

* This was first laid down by Schwedler, "Ueber Nietverbindungen," Dtsch. Bauzeit., 1867, p. 451, etc.

indicated by dotted lines (theoretically, it may even be disadvantageous), but this is, however, often done for the sake of simplicity.

If the necessary number of rivets upon each side of the impact joint is determined, the remarks of Art. 22 hold good for the minimum distance apart in the direction of the force and transverse to it, as well as for the distance from the end of piece. In the grouping, we must have care that, when possible, there may be no greater weakening of the piece by rivet-holes at the place of contact than for the rest of the length. In flanges this can always be observed; it is only necessary that the net cross-section at the outer rivet row *I* of the splice (Fig. 56) shall not be less than that beyond the place of contact, and that the number of rivets per row increase from row to row towards the joint, at the most only by as many rivets as are in row *I*. We are to consider the net cross-section through the innermost rivet row as the effective cross-section of the splice.

In the contact of vertical plates of plate-girders, the weakening due to the rivets is not so important, because we do not need to make the contact when the vertical shear and moments are both simultaneously great; upon other places the vertical plate is always greater than necessary (Art. 17). We should, however, suit the number of rivets to the cross-section of the vertical plate, because a uniform distribution of stress upon this is not to be expected. Since the rivets are double shear, we have, therefore,

$$n_2 = \frac{5}{2\pi d^2} F$$

By the application of this formula, however, we should obtain as the stress upon one rivet

$$N = 2 \frac{\pi d^2}{4} \cdot \tfrac{4}{5} b$$

which gives for slight thickness of the vertical plate, an excessive pressure upon the hole area. Considering this, we can only allow (as in Art. 24)

$$N' = 2 d \Delta b$$

and, hence, for the necessary number of rivets upon each side of the joint,

$$n = \frac{N'}{N} n_1 = \frac{F}{2 d \Delta}$$

or

$$n = \frac{h}{2 d}$$

Here h is the effective height of the vertical plate at the place of union, or the distance between the outer rivet rows, and d is the diameter of rivet.

Even by the most careful arrangement and execution of contact joints, they must be regarded from the above and from the remarks of Art. 19, as in many respects the weak places of the construction. We should, therefore, never allow the pieces which go to make up a construction-piece, such as a flange, to all meet at the same place, but should distribute the joints so far as possible over the entire length. The usual text-books upon bridge construction give sufficient information as to the customary arrangements in such cases.

APPENDIX A.

WE shall now mention the thus far unnoticed efforts with reference to a new method of dimension determination, briefly, but so far as is necessary in order that the reader may be in condition to put them on trial. We shall then conclude with a notice of all efforts thus far made, and a comparison of the allowable stresses per square centimetre as given by the different methods of procedure.

CHAPTER XXVIII.

THE METHODS OF GERBER, MÜLLER, AND SCHÄFFER.

THE first work upon the allowable stress of iron and steel, upon the basis of Wöhler's results, was by Gerber, in 1872, adopted by the Bavarian Government, as "Programm für die Berechnung von Eisenconstructionen," but first published towards the close of 1874.*

The fundamental principle of Gerber may be stated as follows: Any piece of one square unit cross-section may be ruptured by a dead load t. The same effect can be produced by a transitory load of which only one portion c is

* Gerber, "Bestimmung der zulässigen Spannung in Eisenconstructionen," Ztschr. d. Bair. Arch. u. Ing.-Vereins, 1874, p. 101.

constant, and the other d is incessantly repeated. Therefore the strain difference is $\tau\, d$, and we have

$$c + \tau\, d = t = \sigma\, d$$

where σ is a coefficient determined from the above condition.

If, now, we have a piece for which the statical calculation gives a permanent stress B_c and a transitory stress B_v, the combined stress is given by the equation

$$B_c + \tau\, B_v = B_r = \sigma\, B_v$$

reduced to a permanent stress, if only τ or σ is known; and we should have then for the necessary cross-section

$$F = \frac{B_r}{b_r}$$

where b_r is the allowable stress per square unit for *permanent load.* The fictitious strain B_r, Gerber calls the "reduced force."

Since Wöhler has determined the carrying strength t and also for various initial strains c, the differences of strains d for various materials, we can by the insertion of these special values of c, d, t, in the above equation, obtain directly the corresponding values of τ and σ. These coefficients are, however, evidently not invariable, but vary with the ratio

$$\phi = \frac{c}{d} = \frac{B_c}{B_v}$$

In order to obtain the law of this variation, Gerber puts $x = \frac{c}{t}$, $y = \frac{d}{t}$, and determines the curve thus given by the special results of Wöhler's experiments to be nearly a

parabola. He thus obtains a relation, by means of which σ and τ for the varying ratio ϕ are determined and tabulated. We evidently need only tables for σ, since from the above equation $\tau = \sigma - \phi$.

Gerber's formulæ hold good also for alternate tensile and compressive stress, B_c and B_v are always to be inserted with their proper signs (tension positive, compression negative), and therefore ϕ is positive or negative according as B_c and B_v have the same or different signs. We must also remember that B_c is always the stress for the permanent load (dead load for bridges), B_v the stress due to the non-permanent load. If B_v acts opposed to B_c, the total stress may be $B_c + B_v = o$, in which case $\phi = -1$.

The treatise of Gerber gives a table of coefficients σ for iron, from $\phi = o$ to $\phi = +8 \cdot 720$ and $\phi = -9 \cdot 720$, in which σ is considered as having positive sign. But we cannot, as may be seen from the second formula, give to B_r the same sign as B_v. It must take the sign given by $B_c + B_v$. Only when this sum is zero is the sign of B_c applicable also to B_r.

The practical application of Gerber's method takes then the following shape. We determine, in order to take impact into account, *max* B_v with a load $1 \cdot 5$ of the actual, and thus obtain

$$\phi = \frac{B_c}{max\, B_v} \qquad (A)$$

For this value of ϕ we find σ from the table, and thus have

$$B_r = \sigma\, max\, B_v \qquad (B)$$

Finally, we have the necessary effective cross-section

$$F = \frac{B_r}{b_r} \qquad (C)$$

In this formula, Gerber takes for bridge structures, for which especially the greatest durability is required, $b_r = 1600$, and for structures for which lightness is the principal requisite and small changes of form are not prejudicial, $b_r = 2400$.

If the total stress $B_c + B_v$ may be both positive and negative—by B_v we always understand $1 \cdot 5$ times the actual load—ϕ, B_r, and F must be computed for both limiting values of B_v, B_r, and F receive different signs, and the sum of the numerical values of both the F's gives the true cross-section of the piece.

If for a beam p is the dead load, z the $1 \cdot 5$ fold moving load, per running metre, we have when $max\ B_v$ is taken at $1 \cdot 5$ times the moving load

$$\phi = \frac{B_c}{max\ B_v} = \frac{p}{z}$$

The actual maximum stress of a flange is $B_c + \frac{2}{3}\ max\ B_v$, therefore the allowable stress per square centimetre is

$$b = \frac{B_c + \frac{2}{3}\ max\ B_v}{F} = \frac{B_c + \frac{2}{3}\ max\ B_v}{\sigma\ max\ B_v}\ 1600$$

or dividing numerator and denominator by $max\ B_v$

$$b = \frac{\frac{2}{3} + \phi}{\sigma}\ 1600$$

From this formula, the numbers in the first of our tables given in Art. 30 are calculated.*

* The special formula for flanges is not given in Gerber's work. The formulæ are given here because needed for the comparison in Art. 30, as well as to make clear how we have treated some of the doubtful places in his publication. It is there stated, after specifying that $max\ B_v$ is to be found for $1 \cdot 5$ times the moving load, that this value "multiplied by τ" gives the variable in terms of permanent stress. The question arises, What τ? Naturally τ for the

From the above, and by aid of the tables referred to, there will be no difficulty in making use of Gerber's method.

A treatise by Schäffer upon the determination of dimensions was also published in 1874, nearly simultaneously with Gerber's.* Schäffer reversely puts $x = \frac{d}{t}, y = \frac{c}{t}$, agrees as to the already known consequences of Gerber as to the relation between x and y, but deviates essentially in the dimension calculation. His method may be presented as follows:

Let for any piece *max B* be the greatest total stress, the numerical sum being taken, *min B* the least for same signs, and the greatest for opposite signs; then we have, when as above $c + d = a$,

$$\frac{d}{a} = \frac{max\ B - min\ B}{max\ B}$$

where both limiting values are inserted with their signs, so that $\frac{d}{a}$ is always positive.

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corresponding ϕ. But is ϕ to be calculated for the single or 1·5 fold moving load? I have, as seemed proper, assumed the latter. If we calculate ϕ for the actual moving load, we have

$$b = \frac{1 + \phi}{\phi + 1 \cdot 5\ \tau} 1600$$

With this formula I have experimentally computed the numbers for the first table in Art. 30 also, and found, that at the extremes they are equal, and elsewhere give values for b differing at most by 20 kil.

With reference to the determination of dimensions of alternately compressed and extended pieces, Gerber says, finally, "From these reduced forces, with proper regard to their signs, the dimensions may be determined." It is hoped the above presentation will suffice for this.

Schäffer, "Bestimmung der zulässigen Spannung für Eisenconstructionen." Erdkams Ztschr. f. Bauwesen, 1874, p. 398.

Further, if *max* B_v denotes the numerically greatest, *min* B_v the numerically least, stress from the moving load, and B_c that from the dead load, we have in most cases

$$max\ B = B_c + max\ B_v$$
$$min\ B = B_c + min\ B_v$$

This being assumed, we have *min* B_v either zero or of opposite sign from *max* B_v. We have now

$$\frac{d}{a} = \frac{max\ B_v - min\ B_v}{B_c + max\ B_v}$$

and can determine from the relations existing between x and y, and therefore between a and $\frac{d}{a}$ the working strength a for every such relation.

If the cross-section is so chosen that the material is strained by this a per square unit, rupture can just occur. Schäffer seeks to attain the required security by taking the influence of the moving load n fold. Accordingly, the cross-section determination proceeds as follows:

We find from the statical calculation

$$\psi = \frac{n\ (max\ B_v - min\ B_v)}{B_c + n\ max\ B_v} \qquad (A)$$

If ψ is known, we find from the relations between x and y a fictitious working strength (it is not the actual, because ψ is not the actual strain ratio $\frac{d}{a}$)

$$a = \frac{-3\psi + \sqrt{13\psi^2 - 16\psi + 16}}{(2-\psi)^2}\, t \qquad (B)$$

where t is the carrying strength. Finally, we obtain the required cross-section,

$$F = \frac{B_e + n \, max \, B_v}{a} \qquad (C)$$

In order to abridge the calculation, the values of a for all occurring values of ψ may be tabulated.

The greatest stress, which, by this method of calculation, comes upon the square unit, but which is not usually determined, is

$$b = \frac{B_e + max \, B_v}{F} = \frac{B_e + max \, B_v}{B_e + n \, max \, B_v} a$$

For flanges of girders, when the uniformly distributed dead weight and moving loads are p and l, we have

$$b = \frac{p + z}{p + n z} a = \frac{1 + \frac{p}{z}}{n + \frac{p}{z}} a$$

where a is determined by equation (B). Schäffer takes for n the value 3·5 or 4. In Chap. 30 we take the first value.

The work of G. Müller upon dimension calculation* appeared in 1873, soon after that of Launhardt. It is independent of this, as also of the work of Gerber.

Müller proceeds from the assumption, that every stress beyond the elastic limits, therefore producing permanent change of form, if sufficiently often repeated, must cause

* G. Müller, "Zulässige Inanspruchnahme des Schmiedeeisens bei Brückenconstructionen," Ztschr. des Oestr. Ing. u. Arch. Vereins, 1873, p. 197.

rupture. The "primitive safe strength" (I preserve the terms already laid down) u, as the least stress in one direction which can practically produce rupture, is identical with the ordinary elastic limit. For smaller differences of strain, or greater $\frac{c}{d}$, rupture is first possible for the working strength a, and there are, therefore, according to the ratio $\frac{c}{d}$ an indefinite number of elastic limits, varying from u to the carrying strength t.

If we lay off each value of c taken by Wöhler, as abscisas (original compressive strains, negative), and the corresponding value of a as determined by experiment, as ordinate, we have a curve, as shown in Fig. 57. Müller prolonged this curve down to the c axis, and in this manner concludes "from analogies," which, however, are not more minutely specified, as to the value of the primitive safe strength for compression, and thus completes Wöhler's labors.

From this curve, we can determine for every given ratio

$$\phi = \frac{c}{d} = \frac{B_e}{max\, B_v}$$

the working strength a, and then by the application of a suitable coefficient of safety the allowable stress b.

Müller takes $\frac{1}{3}$ as the coefficient of safety, but in $b = \frac{1}{3}\, a$, he already takes into account effects of temperature and rust in the choice of a. The influence of a rise of temperature is taken as equivalent to that stress which would produce the same elongation. It is admitted that temperature influences and the load of a bridge are fortunately never thus properly summed up, but it is, however, claimed that both have a sepa-

rate effect upon the durability, and that "this circumstance evidently requires that the absolute greater stress due to a greater permanent load should be somewhat reduced, because, by the occurrence of additional strains, the danger is increased of reaching the absolute limits of rupture."

Accordingly the ratio $\beta = \frac{a}{u}$ is modified in a way not quite clear, and, to judge from the results, also not quite correct, and determined for successive values of ϕ according to the completed curve obtained from Wöhler's results. We thus obtain

$$b = \tfrac{1}{8}\,\beta\,u$$

Here Müller takes $u = 1600$, and thus there are two tables for allowable stresses, one for tension alone and one for alternate tension and compression, whose application, however, is not to be recommended.

CHAPTER XXIX.

REMARKS UPON ALL THE PRECEDING METHODS.

In the determination of the allowable stress for iron and steel, the chief end is to give *generally*, even if owing to the nature of the case only *approximately*, the relation subsisting between a and the differences of strain. For this, Wöhler's law forms the sole point of departure, since it alone is of general application, and correct for all cases. The special experimental results of Wöhler must indeed be used in the determination of the numerical values of the allowable stress, but with caution, and without assuming their general applicability; just as it has been the habit to look over a list of new experimental results as to the carrying strength t, with approbation indeed, but without, therefore, necessarily making in future exclusive use of the same. If in such tests, for instance, a value of $t = 4000$ were found, the conclusion would be, "good; let us take as a mean 3500." In similar manner, if not even more cautiously, we must proceed with the working strength a. As to the rest, coefficients of safety have been used in the past; they are used to-day, and will also none the less be found necessary in the future.

If, however, Wöhler's law is called in question from no side, and is confirmed anew by the experiments of Spangenberg, it is worthy of question, why a method of calculation is still endured which is recognized as incorrect and danger-

ous. Experiments for the obtaining of further numerical results are certainly very desirable, but have in principle only a significance similar to that which previously was possessed by a new series of experiments upon the ordinary tensile strength; neither Wöhler's law nor the general formulæ based thereon, can thereby suffer any change. We know, indeed, already that every kind of material will furnish somewhat different numerical values, and it is therefore not very plain for just what tests we have still to wait. For special researches upon special bridge-building materials? From all existing experiments, may it not be assumed that for *the same* material smaller differences may be expected than occur between well-recognized *distinct* kinds (Chaps. 5, 12), and that the iron tested by Wöhler was at least not better than that allowable in bridge construction? (13). Or do we wait for tests upon the kinds to-day obtainable? But will these be still obtainable to-morrow? No; a tenable ground against the universal and immediate introduction of a new method of dimensioning has thus far not been advanced; we leave, therefore, the opposition to those whose judgment is swayed by their own conservatism. Shall we, indeed, wait until the new method comes to us from abroad? We have treated here not of theoretical fancies, for this, even to the uninitiated, the names of the distinguished men who have busied themselves with the question, are a sufficient surety. Let coefficients of safety be used at will, still the old methods of calculation are no longer defensible!

Choice still remains between the different proposed changes. If a new method is to find general acceptance, it must be theoretically acceptable, simple in application, and not in conflict with existing experience. It is by no means

proved that the working strength a for every kind of material varies according to the same general law; on the contrary, we can demand that deviations from the general formulæ, founded upon fact, within the limits of variation of the well-known and usual kinds, shall occur. In all these relations, the method of Launhardt is far to be preferred. This view is formed even at first sight, and only confirmed by more careful inspection and by the comparison of practical calculations. After such, I felt first the need of completing and further developing this method, while in the beginning I regarded all methods with equal interest and confidence.

Launhardt's formula in the form (3) is the expression of Wöhler's law itself. By the latter, the limiting values of α are also determined. The single thing arbitrary is the choice of the interpolation formula for α. This choice is surprisingly confirmed by those experiments of Wöhler suitable for testing it (Chap. 3), as well as by other experiments with iron.* Even for more precise determinations than those required here, it would be satisfactory. Further hypotheses and more complicated developments and applications are therefore superfluous.

The idea of Gerber, as presented in Art. 28, is certainly ingenious and clear, but the determination of the relation between a and $\frac{c}{d}$ is somewhat artificial. It was necessary, in order that the formula should also hold good for alternate tension and compression, but the application to this case has certainly not thereby been simplified. Not only must

* Launhardt, "Die Inanspruchnahme des Eisens," Ztschr. d. Hannöv. Arch. u. Ing.-Vereins, 1873, p. 139.

each of equations (A) (B) and (C) be used twice, but the entire method of statical calculation now in use must be modified, and for each piece the influence of the dead weight and the positive and negative maximum effect of the moving load separately determined. This is the case also with the method of Schäffer, while for the application of formula (II) the ordinary method of statical calculation is, as we have seen, all-sufficient.

The manner in which Schäffer estimates the safety of the construction is specially open to objection. Far too much stress is laid upon the influence of the moving and none at all upon that of the permanent load. In fact, Schäffer's formulæ give the safety so much less the greater the permanent load; and when only such a load is concerned, they give as allowable stress the entire carrying strength, $t = 3500$ kil. (Chap. 28. For $B_v = o$, and therefore $\psi = o, a = t$, and $b = a$). As a matter of course, Schäffer's formulæ apply only to bridges for which, indeed, we cannot have permanent loading alone, but a defect in principle is none the less apparent from the above. Gerber allows for the case of permanent load alone, $b = 1600$, which also is not exactly precise.

That Schäffer has not introduced a coefficient of safety for the permanent load, causes also that often alternating stress from tension to compression gives a greater allowable stress than for tension alone. This occurs always when

$$600 < \frac{B_c + max\, B_v}{B_c + n\, max\, B_v}\, a$$

or

$$B_c > \frac{600\, n - a}{a - 600}\, max\, B_v$$

In one case of the second table of Art. 30, this hap-

pens. It is also to be observed that Schäffer's formula (B) for the case of equal alternate tension and compression ($min\ B_v = -\ max\ B_v$, $B_c = o$), by reason of $\psi = 2$, takes the indeterminate form $\frac{o}{o}$.

Müller's method rests in part upon untenable assumptions. Every thing does not justify the assumption that the "rise of temperature has exactly the same influence upon the bridge-piece as a once applied load," but, on the contrary, existing observations contradict it (Art. 10). We have seen that for temperatures from 100° to 200° C. a greater load can be sustained than at ordinary temperatures, although here both influences act together. If, however, the assumption be admitted, it is not correctly applied; for he finds the allowable stress for pieces in alternate tension and compression, in general, greater than for pieces which are only extended and then unloaded. Müller's table 6 is entirely useless; it gives no true representation of the actual change of b, since b cannot increase from $\phi = \frac{B\ comp.}{B\ tension} = 0$ up to $\phi = \infty$, but according to theory and experiment, from $\phi = 0$ to $\phi = 1$, diminishes and first increases for $\phi = \infty$. Thus Müller gives for $\phi = 0$ and $\phi = 1$ respectively, $b = 533$ and $b = 570$, while Wöhler, from whose experiments he takes his departure, gives as the strength in these cases respectively, $a = 2190$ and $a = 1170$. The arbitrary choice of the primitive strength for compression also cannot be recommended. It is permissible to determine the intermediate points of a curve well marked out by characteristic points by interpolation, but not, however, to select these characteristic points themselves.

The methods of Gerber, Schäffer, and Müller have this

in common, that they are all too closely cut out from the numerical results of Wöhler. The last method, deprived of its unnecessary additions, is merely the graphical representation of these results themselves; the latter can only be made practically available by means of tables. Such tables must, to start with, be based upon Wöhler's experiments, but after each future series of experiments, the question must arise whether the old tables are still to be recommended. Formulæ (*I*) and (*II*), on the other hand, are independent of special numerical values, may easily be accommodated to all new results (see also Art. 12), and each one may select his own coefficients of safety.

As regards the application of Launhardt's formula to the deduction of a numerical expression for the allowable stress per square centimetre, it appears to me that even by Launhardt himself the method of taking account of impact is not entirely advantageous. From Wöhler's experiments, Launhardt gets for the working strength of iron, impact disregarded,

$$a = 2190\left(1 + \frac{5}{6}\,\frac{min\ B}{max\ B}\right).$$

The *non* local influence of impact consists now, in the most unfavorable case, first, in an increase of *max B* for which the cross-section is to be proportioned; second, in a diminution of $\frac{min\ B}{max\ B}$, by which the working strength is diminished. The last only is considered by Launhardt when he puts instead of the above value,

$$a = 2190\left(1 + \frac{1}{2}\,\frac{min\ B}{max\ B}\right)$$

In this way, the impact is more regarded the greater *min* B is, and is entirely disregarded when *min* $B = O$, as often happens. This cannot have been intended. Certainly, if we consider tension only (or compression only), a cannot be less than $u = 2190$, but for a piece alternately extended, and then completely or approximately unloaded, there may arise, by reason of impact, in the most unfavorable case, *alternate tension and compression*, so that the working strength should be determined by formula (*II*).

Admitting that AB in Fig. 58 is the curve for working strength, without regard to impact, then, by reason of this last, we should have, according to Launhardt, the line CDE instead of AB, while we should regard the impact throughout, and therefore have FG in place of AB.

CHAPTER XXX.

COMPARISON.

THE following tables give those allowable stresses b, as found by the new method, applicable to such iron construction-pieces as are only extended or only compressed, for which the ratio of the limiting stresses $\phi = \frac{min\, B}{max\, B}$ has the given values. For the special case of bridge-flanges $\phi = \frac{p}{q}$, when p is the dead load, and $q = p + z$ the total load per unit of length. For comparison we have also given the values given by the rule

$$b = \frac{p+z}{p+3z}\, 1600$$

which was used by Gerber in the calculation of the Mainz bridge, and has indeed been still further applied. In the sixth column are the values found from the original expression of Launhardt,

$$b = 800\left(1 + \tfrac{1}{2}\,\frac{min\, B}{max\, B}\right)$$

$\phi = \frac{p}{q}$	$\frac{p}{z}$	Mainz Bridge.	Gerber.	Schäffer.	Launhardt.	Formula 11, Art. 13.
0	0	533	646	600	800	700
$\frac{1}{6}$	0·2	600	740	712	867	758
$\frac{2}{7}$	0·4	659	820	814	914	800
$\frac{3}{8}$	0·6	711	889	910	950	831
$\frac{4}{9}$	0·8	758	947	1000	978	855
$\frac{5}{10}$	1·0	800	997	1088	1000	875
$\frac{6}{11}$	1·2	838	1043	1171	1018	891
$\frac{7}{12}$	1·4	873	1080	1250	1033	904
$\frac{2}{3}$	2·0	960	1172	(1640)	1066	933
1	∞	1600	1600	(3500)	1200	1050

We see here, also, that Schäffer's formula for small permanent load gives too great and for large permanent load gives too little security.

For alternate tension and compression, the allowable stress b is not always, by the methods of Gerber and Schäffer, dependent upon the strain ratio $\phi = \frac{max\,B'}{max\,B}$, but also upon the influence of the dead load B_c; for comparison, therefore, we must take special cases. The stresses in the following table, under *II*, *III*, and *IV*, are for three diagonals in a truss, given in Ritter's "Theorie der Dach- und Brückenconstructionen."

The allowable stresses per square centimetre, according to the American method, alluded to upon page 2, are found as follows. The actual stress b is always

$$b = \frac{max\,B}{F}$$

therefore, by substitution of

$$F = \frac{max\,B + max\,B'}{700}$$

$$b = \frac{max\,B}{max\,B + max\,B'}\,700$$

We have thus found the values given in the table, under the head "American." Compression is minus and tension plus.

	I.	II.	III.	IV.	V.
$max\ B$	arbitrary	$+15380$	$+6230$	$+9550$	arbitrary.
$max\ B'$	0	$-\ 530$	-1280	-4600	arbitrary.
B_e	0	$+\ 2120$	$+\ 710$	$+\ 710$	0
$\frac{max\ B'}{max\ B}$	0	0·034	0·206	0·565	1
Gerber......	646	574	512	437	380
Schäffer.....	600	(609)	542	436	334
Formula (12) Chap. 13..	700	688	628	502	350
American...	700	677	581	472	350
European...	700	700	700	700	700

If, now, any one declines to make use of the advantages which the new method presents as regards the saving of material, that is his affair. The construction will not be any the safer thereby; but he who expends the material is only answerable to him who pays for it. It cannot, however, without danger, be longer allowed to subject pieces which are alternately extended and compressed, and of which an unlimited life is expected, to a stress of 700 kil. per sq. centimetre. He who will go no further, may at least calculate the cross-sections for pieces in alternate tension and compression, as is the practice in America, from the formula

$$F = \frac{max\ B + max\ B'}{700}$$

The stresses thus obtained compare not unfavorably with those obtained from the formula based upon Wöhler's law, as may be seen from the comparison given in the table above.

Plate I.

THE AUTOGRAPHIC TESTING MACHINE.

APPENDIX B.

AUTOGRAPHIC RECORDS

OF THE

STRENGTH AND OTHER PROPERTIES OF MATERIALS.

NOTES BY PROF. R. H. THURSTON.

THE preceding work contains several references to the work of an American investigator—Prof. R. H. Thurston, of the Stevens Institute of Technology.

These references indicate that, in some cases, the author has either misinterpreted the statements made by that writer, or has been misled by errors of translation which may have been introduced into the German reprint of Prof. Thurston's papers. The following extracts from papers read before the American Society of Civil Engineers, and extensively published in the United States and Europe, will exhibit clearly the methods of research, and the results as presented in autographic strain-diagrams.

The accuracy of the work of this machine is evidently determined simply by accuracy of proportion and workmanship, and the fineness of the line of the diagram—conditions entirely under the control of the maker and user.

The formula for tenacity of steel was proposed by Prof. Thurston in 1874, and was intended to represent the strength

of steels untempered but unannealed, and in sections of one square inch or less. It was:

$$T = 60{,}000 + 70{,}000\ C \qquad (1)$$

in which T represents the tenacity in pounds per square inch, and C the percentage of carbon.

For annealed steel the same writer has taken:

$$T = 50{,}000 + 60{,}000\ C \qquad (2)$$

The following are experimental results obtained in the Mechanical Laboratory of the Stevens Institute of Technology, and corresponding estimated resistances, as contributed by him:

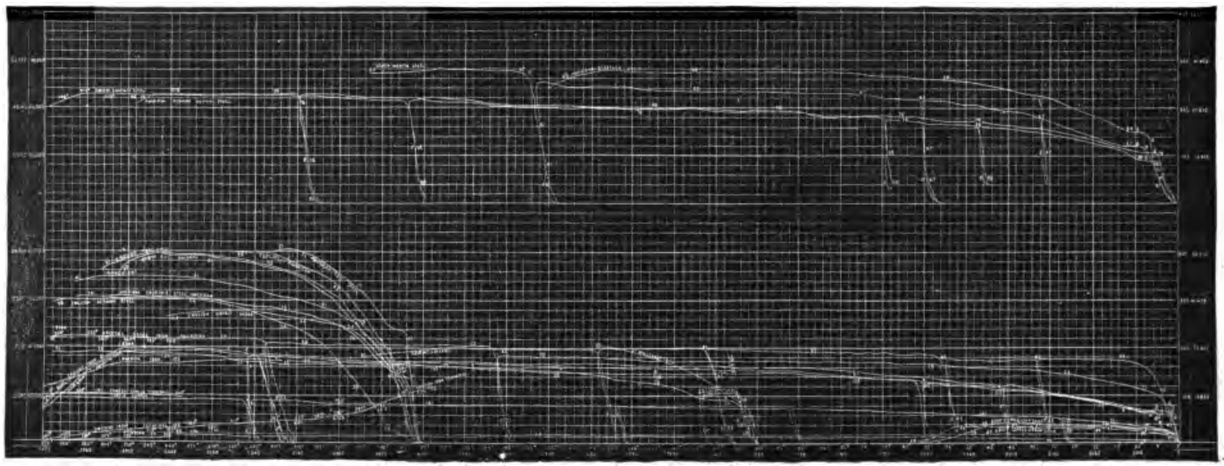
CARBON.	TENACITY BY		CARBON.	TENACITY BY	
	Test.	Calculation.		Test.	Calculation.
0.529	79,062	81,740	1.005	109,209	110,300
0.649	93,404	88,940	1.058	116,394	113,480
0.801	99,538	98,060			
0.867	106,979	102,020			

Plate II.

AUTOGRAPHIC STRAIN-DIAGRAMS OF METALS

PRODUCED BY THE

TESTING MACHINE OF PROFESSOR R. H. THURSTON.



ON THE STRENGTH, ELASTICITY, DUCTILITY, AND RESILIENCE OF MATERIALS OF MACHINE CONSTRUCTION,

And on various hitherto Unobserved Phenomena, noticed during Experimental Researches with a New Testing Machine, fitted with an Autographic Registry.

BY PROF. ROBERT H. THURSTON.

Read February 4, 1874.

SECTION I.

1. INTRODUCTORY.*—Some months ago, while engaged with the advanced classes of the Stevens Institute of Technology, in experimental investigations of the resistance of materials, it was found that coefficients were given, by various authorities, which neither accorded fully with each other or with those then obtained.

The desirability of determining how far these differences were due to errors of observation, and how far to variation in the quality of the materials examined, induced the writer to design several machines for the purpose of conducting with them a more extended and exact series of experiments. The machine for measuring torsional resistance was furnished with an automatic registry, recording a diagram which is a reliable and exact representation of all circumstances attending the distortion and fracture of the specimen. No system of personal observation could probably be

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* *Vide* Journal Franklin Institute, 1873.

devised which could yield results either as reliable or as precise as such a system of autographic registry, and, as no method previously in use had given simultaneously, and at every instant during the test, the intensity of the distorting force and the magnitude of the coincident distortion, it was anticipated that the new method of investigation might be fruitful of new and, possibly, important results. This expectation, as will be seen, has been more than realized.

2. DESCRIPTION OF THE APPARATUS.—The machine, as planned by the writer, and as built in the mechanical laboratory of the Stevens Institute, is shown in Plate I. This form is that with which the investigations to be described were made. Since its construction, in 1872, however, some changes and improvements have been made in the design to adapt it to general work, and new designs have been made for special kinds of work, as for wire-mills, railroad shops, and bridge building.

Two strong wrenches are carried by the frames, and depend from axes which are both in the same line, but are not connected with each other. The arm of one of these wrenches carries a weight at its lower end. The other arm is designed to be moved by hand, in the smaller machines, and by a gear and pinion, or a worm gear in larger forms of the apparatus. The heads of the wrenches are fitted to take the head, on the end of the test-piece.

A guide-curve of such form that its ordinates are precisely proportional to the torsional moments exerted by the weighted arm, while moving up an arc to which the corresponding abscissas of the curve are proportional, is secured to the frame. The pencil-holder is carried on this arm, and as the latter is forced out of the vertical position, the pencil is pushed forward by the guide-curve, its movement being thus made proportionate to the force which, transmitted through the test-piece, produces deflection of the weighted arm. This guide-line is a curve of sines. The other arm carries a cylinder upon which the paper receiving the record

is clamped, and the pencil makes its mark on the table thus provided. This table having a motion, relatively to the pencil, which is precisely the angular relative motion of the two extremities of the tested specimen, the curve described upon the paper is always of such form that the ordinate of any point measures the amount of the distorting force at a certain instant, while its abscissa measures the distortion produced at the same instant.

The convenience of operation, the small cost, and the portability of the machine are hardly less important to the engineer than the accuracy, and the extraordinary extent of information obtainable by it.

3. METHOD OF OPERATION.—The test-piece having been given the shape and size which are found best suited for the purposes of the experiment, and to the capacity of the machine, it is placed in the jaws of the two wrenches, each of which takes one of its squared ends, and, a force being applied to the handle, the strain thrown upon the specimen is transmitted through it to the weighted arm, causing it to swing about its axis until the weight exerts a moment of resistance which equilibrates the applied force. As the magnitude of the distorting force changes, the position of the weight simultaneously changes, and the pencil indicates, at each instant, the value of the stress upon the test-piece. As the piece yields under strains of increasing amount, also, the pencil is carried in the direction of the circumference of the cylinder on which its record is made, and to a distance which is proportional to the amount of distortion; that is, to the "total angle of torsion." As the applied force increases, the specimen yields, and finally, rupture occurring, the pencil returns to the base-line, at a distance from the starting-point which measures the angle through which the test-piece yielded before its fracture became complete.

4. INTERPRETATION OF THE DIAGRAMS.—It has been shown that the vertical scale of the diagrams produced is a

scale of torsional moments, and that the horizontal scale is one of total angles of torsion. Since the resistance to shearing, in a homogeneous material, varies with the resistance to longitudinal stress, it follows that the vertical scale is also, for such materials, a scale of direct resistance, and that, with approximately homogeneous substances, this scale is approximately accurate, where, as here, all specimens compared are of the same dimensions. Since the elasticity of the material is measured by the ratio of the distorting force, to the degree of temporary distortion produced, the diagrams obtained will exhibit the elastic properties of the material, as well as measure its ductility and its resilience.

The initial portion of the line is nearly straight, and the amount of distortion is seen to be approximately proportional to the distorting force, illustrating "Hooke's law," *Ut tensio sic vis.*

After a degree of distortion which is determined by the specific character of each piece, the line becomes curved, the change of form having a rate of increase which varies more rapidly than the applied force. When this change commences, it seems probable that the molecules, which, up to that point, retain generally their original distribution, while varying their relative distances, begin to change their positions with respect to each other, moving upon each other in a manner similar, probably, to that action described by Mon. Tresca, and called the "Flow of Solids,"* and to which attention has already been called by Prof. J. Thompson.†

It is this point, at which the line commences to become concave toward the base, that *is considered to mark the "limit of elasticity."* It is well defined in experiments upon woods, is less marked but still well defined in the "fibrous" irons and the less homogeneous specimens of other metals, and

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* L'Ecoulement des Corps Solides; Paris, 1869, 1871.

† Cambridge and Dublin Mathematical Journal, Vol. III., 1848, pp. 252–266.

becomes quite indeterminable with the most homogeneous materials. This point does not indicate the first "set," since, as will be hereafter seen, a set is found to occur, either temporary or permanent, and usually partly temporary and partly permanent, with every degree of distortion, however small. It is at this "elastic limit" that the sets begin to become considerable in amount and almost wholly permanent.

After passing the elastic limit, the line becomes more and more nearly parallel to the base line, and then, with the woods invariably, and in some cases with the metals, begins to fall rapidly before fracture becomes evident in the specimen. Where the rising portion of the line turns and becomes nearly parallel with the axis of abscissas, the viscosity of the material is such that the outer particles "flow" upon those within, and, while themselves still offering maximum resistance, permit molecules nearer the axis to also resist with approximately maximum force. With the more ductile substances, nearly all are brought up to a maximum in resistance before fracture occurs, and this circumstance will be seen hereafter to have an important influence in determining the resistance to rupture. The hardest and most brittle materials break, with a snap, before any such flow becomes perceivable, and before the line of the diagram commences to deviate, in the slightest degree, from the direction taken at the beginning, and before the approach to the elastic limit is indicated. It is evident that the standard formulas for torsional, as well as for other forms of resistance, cannot be perfectly correct, since they do not exhibit this difference in the character of the resistance offered by ductile and by rigid materials.

The *elasticity* of the material is determined by relaxing the distorting force, at intervals, and allowing the specimen to relieve itself from distortion so far as its elasticity will permit. In such cases, the pencil will be found to have traced a line resembling, in its general form and position, in

respect to the co-ordinates, that forming the initial portion of the diagram,—but almost absolutely straight, and more nearly vertical. The degree of inclination of this line indicates the elasticity, precisely as the initial straight line was made to give a measure of the original stiffness of the test-piece, the cotangent of the angle made with the vertical $Cot.\ \Phi = \frac{1}{Tan.\ \Phi}$ being the ratio of the force required to spring the piece through the range recoverable by elasticity, to the magnitude of that range. The fact, to be shown, that this value is always greater than $Cot.\ \Theta$ for the same metal, is evidence that more or less permanent set will always occur, and that the original stiffness of the specimen is always modified, whatever the magnitude of the applied force. The form of the line of elastic change indicates also the character of the molecular action producing it.

Finally, the form of the curve after passing the maximum, or after passing the point at which fracture commences, exhibits the method of variation of strength during the process of fracture. This portion is very difficult to obtain, with even approximate accuracy, with any but the toughest and most ductile materials. This terminal portion of the diagram would be, theoretically, a cubic parabola, the loss of resisting power varying with the progressive rupture of concentric layers, and the remaining unbroken cylindrical portion becoming smaller and smaller until resistance vanishes with the fracture of the axial line. In some cases, the curves obtained from ductile metals exhibit this parabolic line very distinctly In all hard materials, the jar produced by the sudden rupture of surface particles is sufficient to separate those within, and the terminal line is straight and vertical.

The *homogeneity* of the material tested is frequently hardly less important than its strength, and it is very desirable to obtain evidence which may enable the experimenter to determine the value of tests of samples as indicative of the

character of the lot from which the specimens may have been taken. If the specimens are found to be perfectly homogeneous, it may be assumed with confidence that they represent accurately the whole lot. If the samples are irregular in structure and in strength, no reliable judgment of the value of the lot can be based upon their character, and there can be no assurance that, among the pieces accepted, there may not be untrustworthy material which may possibly be placed just where it is most important to have the best. It is evident that the more homogeneous a material, the more regularly would changes in its resistance take place, and the smoother and more symmetrical would be the diagram. The depression of the line immediately after passing the elastic limit exhibits the greater or less homogeneousness of the material. The fact is illustrated in a striking manner in some of the curves presented, and we thus have—what had never been before found—this method of determining homogeneousness.

The *resilience* of the specimen is measured by the area included within its curve, this being the product of the mean force exerted into the distance through which it acts in producing rupture, *i.e.*, it is proportional to the work done by the test-piece in resisting fracture, and represents the value of the material for resisting shock. The area taken within the ordinate of the limit of elasticity, measures the capacity for resisting shock without serious distortion or injurious set.

The *ductility* of the specimen is deduced from the value of the total angle of torsion, and the measure is the elongation of a line of surface particles, originally parallel to the axis, which line assumes a helical form as the test-piece yields, and finally parts at or near the point where the maximum resistance is formed. Since, in this case, there is no appreciable reduction of section, or change of form, in the specimen, this value of elongation is our actual measure of the maximum ductility of the material, and is an even more

accurate indication than the area of fractured cross-section as usually measured after rupture by tension. It is to be understood that wherever comparisons are here made, without the express statement of other conditions, that specimens of the same dimensions are always represented by the diagrams.

* * * * * * *

6. THE METALS, AND THE CURVES PRODUCED BY THEM. —Plate II. exhibits a series of curves which illustrate well the general characteristics and the peculiarities of representative specimens of the principal varieties of useful metals. In some cases two specimens have been chosen for illustration, of which one presents the average quality, while the other is the best and most characteristic of its class.

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Wrought-iron, as usually made, has a somewhat fibrous structure, which is produced by particles of cinder, originally left in the mass by the imperfect work of the puddler while forming the ball of sponge in his furnace, and which, not having been removed by the squeezers or by hammering the puddle-ball, are, by the subsequent process of rolling, drawn out into long lines of non-cohering matter, and produce an effect upon the mass of metal which makes its behavior under stress somewhat similar to that of the stronger and more thready kinds of wood. In the low steels, also, in which, in consequence of the deficiency of manganese accompanying, almost of necessity, their low proportion of carbon, this fibrous structure is produced by cells and "bubble holes" in the ingot, refusing to weld up in working, and drawing out into long microscopic, or less than microscopic, capillary openings.

In consequence of this structure we find a depression interrupting the regularity of their curves, immediately after passing the limit of elasticity, precisely as the same indication of the *lack of homogeneousness of structure* is seen in diagrams produced by locust and hickory.

The presence of internal strain constitutes an essential peculiarity of the metals which distinguishes them from organic materials. The latter are built up by the action of molecular forces, and their particles assume naturally, and probably invariably, positions of equilibrium as to strain. The same is true of all naturally formed organic substances. The metals, however, are given form by external and artificially produced forces. Their molecules are compelled to assume certain relative positions, and those positions may be those of equilibrium, or they may be such as to strain the cohesive forces to the very limit of their reach. It even frequently happens, in large masses, that these internal strains actually result in rupture of portions of the material at various points, while in other places the particles are either strongly compressed, or are on the verge of complete separation by tension. This peculiar condition must evidently be of serious importance, where the metal is brittle, as is illustrated by the behavior of cast-iron, and particularly in ordnance. Even in ductile metals it must evidently produce a reduction in the power of the material to resist external forces. This condition of internal strain may be relieved by annealing hammered and rolled metals, and by cooling castings very slowly, in order that the particles may assume, naturally, positions of equilibrium. In tough and ductile metals, internal strain may be removed by heating to a high temperature and then cooling under the action of a force approximately equal to the elastic resistance of the substance. This process, called "Thermo-tension," was first used by Professor Johnson in the course of his experiments as a member of a Committee of the Franklin Institute in 1836,* and the effect of this action in apparently strengthening the bars so treated, was stated in the report of the committee. The fact that this effect was very different with different kinds of iron was also noted ; but it does not appear that the

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* Journal Franklin Institute, 1836–7.

cause of this, which they term "an anomalous" condition of the metal, was discovered by them.

Metals which are very ductile may frequently be relieved of internal strain, also, by simply straining them while cold to the elastic limit, and thus dragging all their particles into extreme positions of tension, from which, when released from strain, they may all spring back into their natural and unstrained positions of equilibrium. This fact, which does not seem to have been previously discovered by investigators of this subject, will be seen to have an important bearing upon the resisting power of materials, and upon the character of all formulas in which it may be attempted to embody accurately the law of resistance of such materials to distorting or breaking strain.

Since straining the piece to the limit of elasticity brings all particles subject to this internal strain into a similar condition, as to strain, with adjacent particles, it is evident that indications of the existence of internal strain, and through such indications a knowledge of the value of the specimen, as affected by this condition, must be sought in the diagram, before the sharp change of direction which usually marks the position of the limit of elasticity is reached. As already seen, the initial portion of the diagram, when the material is free from internal strain, is a straight line up to the limit of elasticity. A careful observation of the tests of materials of various qualities, while under test, has shown that, as would, from considerations to be stated more fully hereafter, in treating of the theory of rupture, be expected, this line, *with strained materials, becomes convex towards the base line*, and the form of the curve, as will be shown, is parabolic. The initial portion of the diagram, therefore, determines readily whether the material tested has been subjected to internal strain, or whether it is homogeneous as to strain. This is exhibited by the *direction* of this part of the line as well as by its form. The existence of internal strain causes a loss of stiffness, which is shown by the deviation of this part of the line from the vertical to a degree

which becomes observable by comparing its inclination with that of the line of elastic resistance, obtained by relaxing the distorting force—that is, the difference in inclination of the initial line of the diagram and the lines of elastic resistance, *e*, *e*, *e*, indicates the amount of existing internal strains.

*   *   *   *   *   *   *

11. General Conclusions.—These diagrams, which are the autographs of all the useful metals, illustrate sufficiently well the remarkable fulness and accuracy with which their properties may be graphically represented, and the convenience with which they may be studied, with the aid of so simple a recording machine. A comparison of results deduced as shown, with those obtained, so far as they can be obtained at all, by the usual method of simply pulling the specimens asunder, and trusting to, sometimes, unskilful hands and an untrained observer, for the adjustment of weights and the registry of results, will indicate the close approximation of this method in even ascertaining the behavior of the metal in tension. On examining the beautifully plotted curves given by Knut Styffe, as representing the results of the experiments, made by him and by his colleagues, with a tensile machine, no one can fail to be struck with the similarity of those diagrams to the curves here produced automatically, and it will be readily believed that not only must there be very perfect correspondence of results where the two methods are carefully compared, but, also, that any theory of rupture must be defective which does not apply to both cases. The equations of the curves here given and those of the curves obtained by Styffe must have forms as similar as the curves themselves.

12. Testing within the Limit of Elasticity.—In determining the value of materials of construction, it is usually more necessary to determine the position of the limit of elasticity and the behavior of the metal within that limit than to ascertain ultimate strength or except, perhaps, for machinery, even the resilience. It is becoming well recognized by engineers who are known to stand highest in

the profession, that it should be possible to test every piece of material which goes into an important structure and *to then use it* with confidence that it has been absolutely proven to be capable of carrying its load with a sufficient and known margin of safety. It has quite recently become a common practice to test rods to a limit of strain determined by specification, and to compel their rejection when found to take a considerable permanent set under that strain. The method here described allows of this practice with perfect safety. The limit of elasticity occurs within the first two or three degrees, and, as seen, the specimen may be twisted a hundred or even sometimes two hundred times as far without even reaching its maximum of resistance, and often far more than this before actual fracture commences. It is perfectly safe, therefore, to test, for example, a bridge-rod up to the elastic limit, and then to place the rod in the structure, with a certainty that its capacity for bearing strain without iniury has been determined and that formerly existing internal strain has been relieved. The autographic record of the test would be filed away, and could, at any time, be produced in court and submitted as evidence—like the "indicator-card" of a steam-engine—should any question arise as to the liability of the builder for any subsequent accident, or as to the good faith displayed in fulfilling the terms of his contract.

13. The above will be sufficient to show the use and the value of this method. In the course of experiment upon a large number of specimens of all kinds of useful metals and of alloys, a number of interesting and instructive researches have been pursued, and some unexpected discoveries have been made.

THE STRENGTH AND OTHER PROPERTIES OF MATERIALS OF CONSTRUCTION,

As deduced from Strain-Diagrams automatically produced by the Autographic Recording Testing Machine.

BY PROF. ROBERT H. THURSTON.

Read December 31, 1875.

IN a paper read before the Society of Civil Engineers in February and April, 1874,* the writer gave an account of a series of researches which he had made with a novel form of apparatus, and illustrated the work by *fac-similes* of a collection of automatically produced strain-diagrams. The new method of investigation adopted and the importance of some of the conclusions deduced from the autographic records have attracted much attention and the paper has been extensively republished.† It has recently been translated into the German for Dingler's Polytechnisches Journal, and its publication has been followed by a paper by a distinguished colleague of the writer, Prof. Kick,‡ of the Institute of Technology at Prague, who makes a number of criticisms§ which indicate that it may be advisable to consider some of the more obscure points in the original paper at greater length and to exhibit the sources of the errors which have been committed by the critic.

The first criticism made by Prof. Kick, as will be seen

* Transactions, Vol. II., page 349; Vol. III., page 1; † Journal of the Franklin Institute, 1874; Van Nostrand's Mag., 1874; Dingler's Polytechnic Journal, etc., etc., 1875; ‡ International Jury, Vienna, 1873; § Kritik über R. H. Thurston's Untersuchungen über Festigkeit und Elasticität der Constructionsmaterialen, von Friedrich Kick, Bd. 218, H. 3.

by a perusal of his paper, is a statement that important discrepancies exist between the results obtained experimentally by the author of the criticism and by the writer. This difference is attributed to an assumed peculiarity of the apparatus and of the method of experiment adopted by the writer, which is asserted to produce serious errors. That such a difference does appear between the results obtained by the writer and the critic is undoubtedly the fact; that they are attributable to the cause assigned is less evident, and what follows may prove the assertion entirely unfounded. The critic makes an assumption of faulty manipulation without evidence of its existence, and then claims to "prove" mathematically that the apparatus, which is asserted to be "dynamic" in its action, records its results statically and thus introduces fatal errors of record.

The mathematical portion of the paper is correct, and we will take advantage of that fact and will show how far the adverse element—the resistance due to the acceleration of weight — which is so boldly asserted to be the cause of "serious" errors, is likely to introduce such errors.

Taking an extreme case: Supposing a perfectly rigid test-piece to be under test, the velocity of motion of the weight would be precisely equal to that of the handle, and would be a *maximum*. Actually, the test-piece always yields and the velocity of the weight is invariably less than that of the handle. In the greater number of cases, the weight moves with much less rapidity than the handle, even when moving at its highest rate of speed, and, during the greater part of the test, the rate of motion of the weight is so slow as to be imperceivable and incapable of measurement, and, at other times, the weight actually moves slowly downwards, as is seen by reference to the published strain-diagrams, on which the relative motion of weight and handle can be readily determined.

The motion of the weight is, in fact, independent of that

of the handle and varies with the resistance of the test-piece, rising or falling as that resistance increases or diminishes, always slowly and almost invariably with very much less velocity than that of the handle. In making tests with this machine, the handle is always moved very slowly, and when attempting to secure diagrams for scientific purposes especial precaution is taken.

The following figures represent the rate of motion of the *handle*, measured alternately for a somewhat rapid and for an ordinarily slow motion. The motion of the weight is, as has been shown, very much slower.

	TIME.	ANGLE.	R. COS.	SPACE.	MAX. MOMENT.
(*A*)	1 Min.	16°.00	47.125 in. 1.197 m.	13.54 in. 0.362 m.	135.987 ft. lbs.
(*B*)	2 Min.	37°.66	38.75 in. 0.984 m.	32.00 in. 0.813 m.	292.755 "
(*C*)	1 Min.	16°.00	47.125 in. 1.197 m.	18.54 in. 0.362 m.	135.987 "
(*D*)	1 Min.	37°.06	39.00 in. 0.996 m.	31.70 in. 0.805 m.	291.70 "

Where the effort was made to attain greater rapidity of motion, the following results were obtained:

	TIME.	ANGLE.	R. COS.	SPACE.	MAX. MOMENT.
(*E*)	1 Min.	33°.66	40.75 in. 1.035 m.	28.78 in. 0.761 m.	267.02 ft. lbs.
(*F*)	1 Min.	47°.66	33.00 in. 0.838 m.	40.63 in. 1.032 m.	350.98 "

Prof. Kick states correctly the resistance due to acceleration of the motion of the weight as equal to $\frac{vG}{gt}$, and the total amount of stress as

$$S = G + \frac{vG}{gt} \qquad (1)$$

by a perusal of his paper, is a statement that important discrepancies exist between the results obtained experimentally by the author of the criticism and by the writer. This difference is attributed to an assumed peculiarity of the apparatus and of the method of experiment adopted by the writer, which is asserted to produce serious errors. That such a difference does appear between the results obtained by the writer and the critic is undoubtedly the fact; that they are attributable to the cause assigned is less evident, and what follows may prove the assertion entirely unfounded. The critic makes an assumption of faulty manipulation without evidence of its existence, and then claims to "prove" mathematically that the apparatus, which is asserted to be "dynamic" in its action, records its results statically and thus introduces fatal errors of record.

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Prof. Kick states correctly the resistance due to acceleration of the motion of the weight as equal to $\frac{vG}{gt}$, and the total amount of stress as

$$S = G + \frac{vG}{gt} \qquad (1)$$

in which expression, $S =$ the total stress, $v =$ the acquired velocity at the end of the time t, $G =$ the weight and $g =$ the acceleration of gravity $= 32\frac{1}{6}$ feet $= 386$ in. $= 9.8$ m.

$$\frac{S}{G} = 1 + \frac{v}{gt} \qquad (2)$$

Then, for the several cases just given, assuming the velocities to be those of the weight, as improperly asserted by Prof. Kick, we get:

$$\text{(A.)}\ \frac{S}{G} = 1 + \frac{v}{gt} = 1 + \frac{13.54 \times 2}{386 \times 60} = 1.001212;$$

$$\text{(B.)}\ \frac{S}{G} = 1 + \frac{32 \times 2}{386 \times 120} = 1.001401;$$

$$\text{(C.)}\ \frac{S}{G} = 1 + \frac{18.54 \times 2}{386 \times 60} = 1.001212;$$

$$\text{(D.)}\ \frac{S}{G} = 1 + \frac{31.70 \times 2}{386 \times 120} = 1.001347;$$

And for those cases in which the rate of acceleration was made as great as could be obtained by the exertion of all the strength of the operator:

$$\text{(E.)}\ \frac{S}{G} = 1 + \frac{28.78 \times 2}{386 \times 60} = 1.002481;$$

$$\text{(F.)}\ \frac{S}{G} = 1 + \frac{40.68 \times 2}{386 \times 60} = 1.006301.$$

It is seen from the above that the maximum possible errors, due to the cause assumed by the critic as the source of the discrepancies which he has found to exist between his work and the self-recorded results given by the autographic machine, are necessarily some fraction of one eighth of one per cent. Every experienced investigator in this depart-

ment of scientific research knows, however, that this limit of error falls far within the limit of variation of quality of every material of construction, even when nominally of the same grade. The criticism is therefore seen to have no practical weight.

Now, determining the relative motion of handle as given above, and of the weight, from the strain-diagrams published, and taking wrought-iron as the best illustrative example, it will be seen that, within the elastic limit, the error claimed to destroy the value of the data secured may possibly amount to 0.001, and that at the limit of elasticity even this error entirely disappears, since the weight there ceases rising. Beyond the limit of elasticity, the error is that due to a rise of the weight equal to an exceedingly minute fraction of the motion of the handle, and is so small that it would be quite impossible to detect it on the diagram by any method of measurement in use. The criticism of the distinguished author of this "*kritik*" is thus seen to be quite insufficient to account for the discrepancies noted by him. He is quite right in looking for the source of error in the machine—provided that the results of the writer are erroneous and those of Prof. Kick are right; for, in the former, the story is told by the machine itself, and cannot be attributed to errors of personal observation, as may those existing in data acquired by the older methods of research.

It may be safely asserted that the errors due to the inertia of the weight and to its acceleration may, by careful handling, be made as minute and as practically immeasurable as those due to the same cause in the older forms of testing machines. Considering the facts, that the results obtained by the older methods of testing are liable to errors arising from personal observation, while in the method adopted by the writer in the autographic recording testing-machine they are automatically registered, it would seem that the advantage, in respect to accuracy, must be on the side of the new method.

The writer believes the facts exhibited above to prove conclusively that the bold assertion of the foreign critic—that with the greatest care these strain-diagrams are liable to be incorrect and untrustworthy—is without real basis, and is itself absolutely incorrect.

The second criticism of Prof. Kick, in which he suggests this assumed source of error to be the cause of the differences in the initial portions of diagrams (6, 101, and 85 of Plate II., Vol. II., page 378), which the writer attributed to peculiar conditions of molecular or mechanical structure, is not only invalidated by what has been shown above, but most conclusively by a large number of experiments made before and after the date of the original paper, in which the noted peculiarities were very marked, although the experiments were conducted with uniform precaution. The fallacy of the criticism is still further proven by the characteristic differences noted in the initial portion of the diagram where different metals are compared, as shown in the published diagrams of iron, steel, copper, tin, etc. Such differences could not possibly arise from the assumed cause.

Professor Kick adduces as what he asserts to be "absolute proof" of the existence of the source of error above alluded to, the peculiar strain-diagrams, 101 and 118. These show the rapid motion of the handle (not of the weight) to be followed by a fall of the weight and a drop of the pencil. This was attributed by the writer to a weakening of the metal by rapid distortion; a conclusion which has been confirmed by a study of Kirkaldy's experiments with his tension apparatus, by many experiments since made by the writer with the autographic machine, by numerous experiments made by Com. Beardslee at the Washington Navy Yard with a tension machine having peculiar facilities for exhibiting this phenomenon, and especially by the experiments made on a very large scale on iron beams for targets, as described by Gen. Barnard in a paper read before the Society (Transactions, Vol. I., page 173), and referred to by

the writer in the discussion at the Seventh Annual Convention (Vol. III., page 128).

The error into which the critic has fallen will be seen at once when it is noted that during this rapid motion of the handle and the distortion of the test-piece, produced by a heavy blow on the handle, the weight had no time to move and the drop of the weight succeeded the distortion, as is explicitly stated in the original paper to be an evidence of the weakening which is a consequence of rapid distortion. This evidence would seem to be quite sufficient. But the experiment described by Gen. Barnard, to say nothing of those of Com. Beardslee, are certainly conclusively corroborative.

The exception taken by the critic to the principles (6) and (7) are fully met by the above, and no more need be written on this point.

In the paper here referred to, Prof. Kick goes on to state that the phenomenon of "elevation of the elastic limit by strain," claimed to have been discovered by the writer, was discovered by General Uchatius of Vienna, and published in April, 1874. (*Die Stahlbronze. Vortrag gehalten am* 10. *April*, 1874, *Wien.*)

The writer is greatly pleased to find his work confirmed by so distinguished an authority, but his own discovery of this remarkable and important phenomenon was made months earlier, and was announced at the Annual Meeting of this Society, November, 1873, and formally placed on record in a "Note on the Resistance of Materials," read November 19th, 1873. (Transactions, Vol. II., page 239.) The phenomenon was also discovered by Com. Beardslee, U. S. Navy, soon after, and by an entirely independent method of investigation, and was made known by him before the end of that year. It has since been observed by many experimenters, but the writer has as yet met with no claim of priority of discovery.

Prof. Kick asserts that the extensions estimated by the

writer cannot be correct, because of a diminution of length in the specimen, and because of the influence of the cohesion existing between the inner and outer fibres of the mass. The writer can only say that experiment does not seem to confirm these assumptions and assertions.

In regard to the elongations given by the writer, amounting, with some ductile materials, to 120 per cent, it need only be repeated that it was explicitly stated that those figures are given as the best indication of the ductile quality of the material, that they are proportional to the maximum elongation of the most extended portions of the metal tested by tension, for the very reason stated in opposition by Prof. Kick, that the tension specimen invariably "necks down," and does not stretch as a whole, or uniformly; and it was stated that these factors of extension are related to the reduction of cross-section observed in tension, and are such as do occur within the elastic limit in homogeneous materials, and such as would be observed were the material under tension to draw down uniformly from end to end until fracture occurs, leaving the whole piece, in that case, of the diameter of the fractured section actually observed in the tension experiments.

The writer has stated his idea that the reduction of section by tension and not the extension of the whole specimen, is the most accurate measure of the ductility of the material. After passing the elastic limit, and after "necking down" begins, the elongation of a test-piece under tension is a function of its diameter and not of its length; and the whole extension may be expressed by the formula, $E = A\,l + B f d$, an expression which the writer has not yet met with in any work on this subject.

The writer has noticed these errors of the critic with as much surprise as regret, and especially as he finds them associated with the very excellent caution against "roaming in the fields of conjecture" in such scientific work.

Finally, comparing conclusions (10) and (11) with (19) and (20?) in which the effects of temperature are referred to, the critic notes an apparent discrepancy which a more careful reading would have explained and the necessity of reference to them perhaps not have arisen. It is not, however, impossible, that the writer was not sufficiently explicit. Referring to the original paper, it will be seen that the author quotes from an earlier monograph on the effects of temperature, in which all of the earlier researches of both physicists and engineers, so far as they were accessible to him, were collated, and the conclusions, derived by comparison, were that a rise of temperature decreases the resisting power of materials while increasing their ductility and sometimes their resilience; a decrease of temperature seemed to produce the opposite effect. The generally conflicting testimony of those who, on the one hand, had experimented by steady stress, and those who, on the other hand, had experimented by shock, thus seemed to be reconcilable. The apparent discrepancies between authorities were concluded to be due to differences of method similar to those which are claimed by Prof. Kick to distinguish the researches of the writer from those of the better known authorities—but with more reason.

Subsequently the invention of the autographic testing-machine having, for the first time, furnished a means of making simultaneous determinations of the several mechanical properties of the test-piece, the real facts seemed to be proven to be slightly different, and as stated in (20) that with pure well-worked metals the principle enunciated in (28) is fully illustrated, and a decrease of temperature is accompanied by an increase of strength, ductility, and resilience; (21) that materials which are impure and irregular in character may exhibit exceptions to and even reversals of that principle in changes of ductility, and, while increasing in power of resisting simple stress, may, by a diminution of temperature, lose their power of resisting shocks; and that the effect of

change of temperature probably varies with the character of the material.

The writer is grateful for the pleasant compliment contained in the closing paragraph of the paper of Prof. Kick, and trusts that the above remarks may indicate that the ordinarily useful work of the confessedly valuable addition to "practical" testing apparatus, which has been found in the autographic recording testing machine, may prove to be supplemented by not less valuable scientific work.

NOTE

ON THE RESISTANCE OF MATERIALS,

AS AFFECTED BY FLOW AND BY RAPIDITY OF DISTORTION.

THE effect of the "Flow of Metals" and of the force of polarity described by Prof. Henry, in modifying their resistance to external stress and their strain, was alluded to by the writer in preceding Transactions, as follows: *

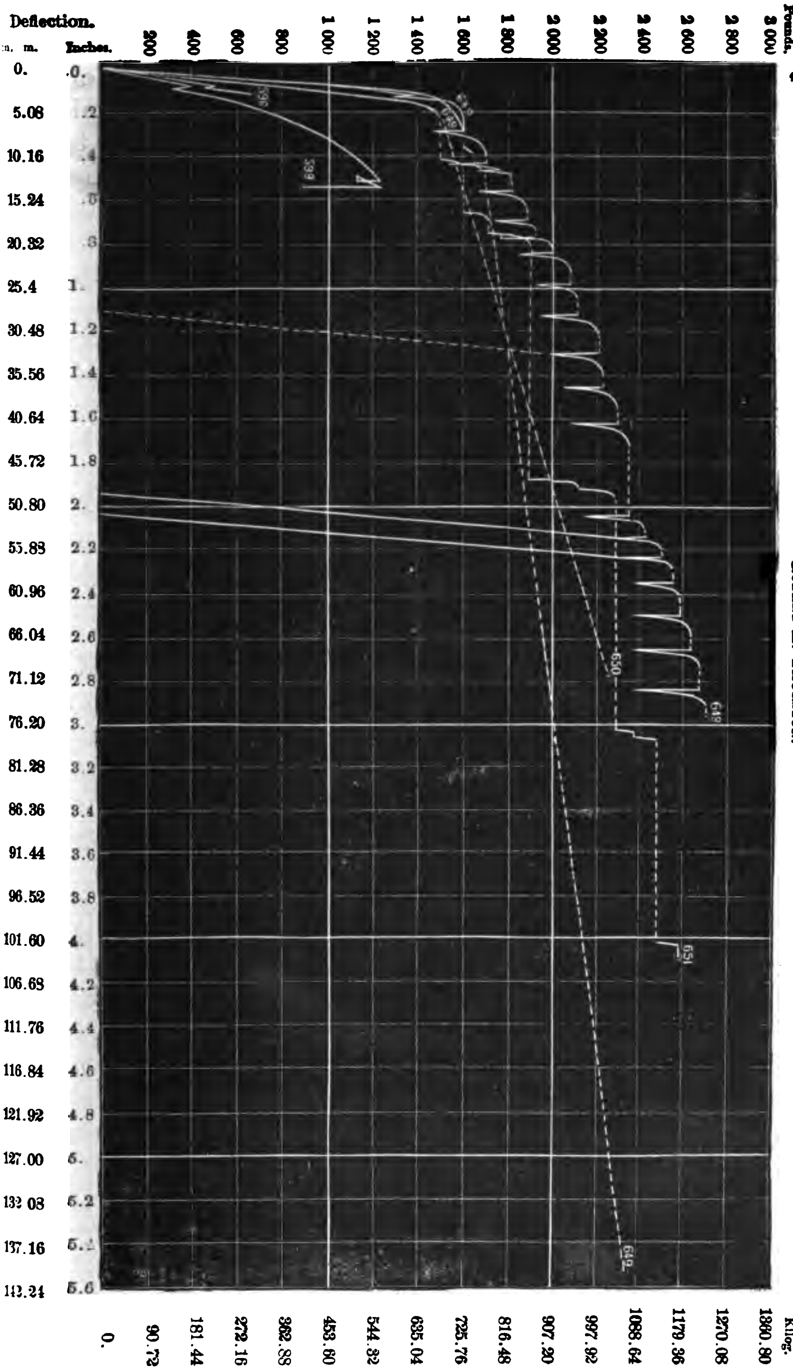
"The same molecular movement, or flow, which rearranges the internal force and relieves internal strain, may be a phase of that viscosity which Vicat supposed might in time permit rupture of metal subjected to stress nearly approaching its original ultimate resistance, the one action being a more immediate result than the other, and the latter producing its effect, even when cohesive force may have been actually intensified."

* LXXXII. On the Mechanical Properties of Materials of Construction. Vol. III., page 13.

STRAIN DIAGRAMS

OF FOUR BARS OF WROUGHT IRON, AND TWO OF COPPER-ZINC ALLOYS TESTED BY TRANSVERSE STRAINS.

ROBERT H. THURSTON.



It was noted, however, that, in all cases in which wrought iron and steel had been subjected to stress exceeding the elastic limit, the metal had exhibited no tendency to flow, and that, in nearly every case observed, an actual "elevation of the elastic limit by strain" had taken place. No experiment had then been made by the writer in which the same sample had exhibited both the elevation of the elastic limit by strain and the phenomenon of flow.

Since that time, when experimenting upon copper, strain-diagrams produced automatically have been observed to exhibit this double effect. The elevation of the elastic limit has occurred in the earlier part of the test, and, at a later period, the strain-diagram exhibits flow, the metal yielding under a gradually decreasing stress. The progressive distortion which had never been observed by the writer in iron or steel, has, since the date of the paper, been frequently noted in other materials.

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Tests by tension of copper-tin alloys exhibit similar results, and these observations and experiments thus seem to confirm the remarks of the writer as above quoted, and to indicate that, under some conditions, the phenomena of flow and of elevation of the elastic limit by strain may be coexistent and that progressive distortion may occur with "viscous" metals.

The paper referred to enunciated a principle which had been deduced from experiments on wrought-iron which is, if possible, of more vital importance to the engineer than the facts just given—namely, "That the time during which applied stress acts, is an important clement in determining its effect, not only as an element which modifies the effect of the *vis viva* of the attacking mass and the action of the inertia of the piece attacked, but, also, as modifying seriously the conditions of production and relief of internal strain by even simple stresses." *

* Vol. III., page 30.

It was then shown, by autographic strain-diagrams, that some materials yield the more readily the more rapidly the distortion and rupture are produced, their resistance varying in some inverse ratio with the rapidity of change of form. It was further suggested that this action might be closely related to the opposite phenomenon of the elevation of the elastic limit by strain. An explanation was offered in the theory that, with rapid distortion, insufficient time is allowed for the relief of internal strain in materials capable of exhibiting that condition. It was further remarked that "the most ductile substances may exhibit similar behavior, when fractured by shock or by any suddenly applied force, to substances which are comparatively brittle," and illustrations were given of such behavior, and the precautions to be taken by the engineer, in view of this important modification of the resistance of materials by velocity of rupture, were stated.

The writer has continued his experimental researches, with occasional interruption, since that time, and has found the above given statements confirmed, and that relations exist between these phenomena of strain and the time under stress, which may properly be stated here as complementary of the principles already published in the two preceding notes which have appeared in the Transactions.*

Should it be true, as suggested by the writer, that the cause of the decreased resistance, sometimes observed with increased velocity of distortion, is closely related to the cause of the elevation of the elastic limit by strain,† it would seem a simple corollary, that *materials so inelastic and so viscous as to be incapable of becoming internally strained during distortion should offer greater resistance to rapid than to slowly produced distortion*, in consequence of their inability to "flow" so rapidly as to reduce resistance by such

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* LXI. Vol. II., page 239. CXV. Vol. IV., page 334.

† Transactions, Vol. III., page 363.

fluxion at the higher speed, or by correspondingly reducing the fractured section. This principle has been shown, by a large number of experiments, to be frequently, if not invariably, the fact. Copper, tin, and other inelastic and ductile metals and alloys are found to exhibit this behavior, and are, therefore, quite opposite in this respect to ordinary wrought-iron and worked steel.

The writer has noted the fact that very soft wrought-iron does not always exhibit an observable elevation of the elastic limit by strain, and Com. L. A. Beardslee, U. S. N., has recently observed that the softest and most ductile specimen of iron yet tested by him at the Washington Navy Yard exhibited a perceptible increase of resistance with a considerable increase of rapidity of extension.* This metal was peculiar in its softness and extreme extensibility. All the irons of commerce appear to belong to the other class.

The records of the Mechanical Laboratory of the Stevens Institute of Technology frequently illustrate the proposition that metals which gradually yield under a constant load offer increased resistance with increased rapidity of rupture.

The curves of deflections of a considerable number of ductile metals and alloys are very smooth when the time during which each load has been left upon them is the same; but, whenever that time has been variable, the curve has been irregular. Bars of such metals broken by transverse stress give a greater resistance to rapidly increasing stress than to stress slowly intensified. Two pieces of tin from the same bar were broken by tension, the one rapidly and the other slowly. The first broke under a load of 2100 and the latter of 1400 pounds. The example illustrates well the very great difference which is possible in such cases, and seems, to the writer, to indicate the possibility in extreme cases of obtaining results which may be fatally deceptive when the time of rupture is not noted.

* It follows that the strain-diagrams may indicate the *purity* of irons tested as well as of their mechanical properties. [Note by Prof. Thurston.]

Autographic strain-diagrams, given by this class of metals, exhibit smooth, straight and horizontal lines for long distances on the paper where the distortion is produced by a uniform motion. Increasing the rapidity of distortion causes an immediate and sustained elevation of the pencil, and a decrease of velocity causes the line to droop to a lower level. In some experiments a torsion of one revolution in a half-hour, the test-piece being $\frac{5}{8}$ inch diameter and one inch long, just kept the pencil on a horizontal line.

Two test-pieces from the same bar were broken, the one rapidly, the other slowly. The former gave a strain-diagram of which the maximum ordinate was about 50 foot-pounds higher than the maximum of the latter, the difference being nearly 50 per cent of the higher.*

It is evident that, whatever the character of the material and whatever the velocity of rupture, the effect of the *inertia* of the mass, and of particles not immediately affected by a shock, remains, and that its effect is *always* to reduce the resilience of the metal and its resistance to shock; and this reduction may, in many cases, more than compensate the increase of resistance here noted. Its tendency is always to produce a sharp fracture which, with such sudden blows as are given by cannon shot, for example, may resemble the break characteristic of brittle and non-ductile substances.

The writer would, therefore, divide the metals used in construction into two classes:

1*st. Metals subject to internal strain by artificial manipulation and which may exhibit an elevation of the elastic limit by strain, and decreased power of resisting stress under increasing rapidity of distortion. The ordinary irons of commerce are typical of this class.*

2*d. Metals of an inelastic viscous character, not subject to internal strain and not usually exhibiting an elevation of the*

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* The inertia of the *weight* in these examples had no measurable effect in modifying those results.

elastic limit by strain, and which offer increased resistance when the rapidity of distortion is increased. Tin is a typical example of this class.

It is obvious that the value of the former class for the construction of the engineer is vastly greater than the latter, and especially for permanent loads and low factors of safety.

The depression of the elastic limit has been observed previously in materials, but less attention has been paid to it than the importance of the phenomenon would seem to demand. The accompanying plate exhibits the strain-diagrams produced by plotting the results of experiments. They are selected as typical examples, and as representing the two classes of materials described.

In making the experiments the bar was mounted on cylindrical steel bearings, which were themselves supported on accurately planed level surfaces, and the deflection was produced by means of a powerful screw and a large hand-wheel. The weight was measured by a Fairbanks scale combination, and the deflections and sets by a special measuring apparatus which reads to 0.0001 inch, with an error of 0.000741. Touch is indicated by a delicate Stackpole level. The measuring instrument was unaffected by the forces tending to distort the straining apparatus. The deflecting force was adjusted by the scale-beam. The bar being in place, the weight to be put on it was set off on the scale-beam, and the screw was carefully turned until, by its pressure on the middle of the bar, the scale-beam slowly rose and vibrated about the middle of its range, which point was indicated by a pointer at the end of the beam, traversing a fine lined scale on the frame. When the adjustment had become satisfactory, the deflection was read off and the beam usually released, in order that the set might be observed. It was then again deflected by a heavier weight. Occasionally the bar was left thus strained, and with a constant deflection, for a considerable period of time, and the change of effort exerted by it noted at frequent intervals.

In all such cases the scale-beam gradually drooped, and a decreased effort to effect restoration of form was indicated. When the beam had fallen, the weight was pushed back until the beam arose and vibrated about the centre line again, and the weight and time were recorded. This was repeated as the beam exhibited less and less loss of power of restoration, and when this decrease of effort no longer exhibited itself, a new series of deflections was produced.

The bar No. 599—zinc 90, copper 10—which was quite ductile, exhibited an unchanged law of relation of amount of deflection to intensity of deflecting force, and, as shown by the diagram, the curve representing its test pursued the same general direction after one of these "time-tests" as before.

The loss of effort at 163 pounds is seen to have been about 20 pounds, the deflection amounting to 0.0347 inches, and the effort falling from 163 to 143 pounds. At 403 pounds the loss of restorative force is about the same; the figures fall from 403 to 333 pounds, the deflection being held constant at 0.0886 inches, again from 333 to 302 pounds at a deflection of 0.0896, and still again from 1233 to 1137 pounds at a deflection of 0.5209 inches.

Before the bar, under further deflection, had quite regained its original resisting power, the "time-test" was repeated, the deflection amounting to 0.5456 inch, and the weight applied being 1233 pounds. The result noted was quite unanticipated. The effort steadily decreased at a varying rate, which is indicated by the diagram of time and loads, and the bar finally snapped sharply, and the two halves fell upon the floor. The effort had decreased to 911 pounds. The deflection was precisely what it had been under the load of 1233 pounds. The beam had balanced at 911 pounds for about three minutes when the fracture took place. An assistant was sitting fifteen or twenty feet from the machine at the instant, but no one had approached the machine after the last adjustment of the weight.

This is a case without parallel in the experience of the writer, and its conclusion indicates a possibility of depreciation in resisting power in the class of metals of which tin has been taken as the type, which depreciation, in the present state of our knowledge of the properties of such metals in this regard, it may be safest to assume to be a source of danger in some cases in which the load approaches the maximum resisting power of the piece. This illustrates the case of progression of flow until the section most strained has been weakened to the point of actual molecular disruption, which disruption would seem to have been here produced by the effort of other and less injured portions to resume their original positions, and to straighten the two halves of the bar. It would seem that such action should be determined by flow occurring in a somewhat ductile but still somewhat elastic metal.

The strain-diagram of this bar is seen to be nearly hyperbolic; but the law of Hooke, *ut tensio sic vis*, holds good, as usual, up to a point at which the load is about one-half the maximum. The curve of times and loads exhibits the rate of loss of effort while the bar was finally held at a deflection of 0.5456 inch, the load being carefully and regularly reduced, as the effort diminished, from 1233 to 911 pounds, at which latter figure the bar broke. The curve is a very smooth one.

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An example of somewhat similar behavior was exhibited by a metal of very different quality.

This bar—No. 596, zinc 75, copper 25—was hard, brittle, and elastic, but must apparently be classed with tin in its behavior under either continued or intermitted stress.

There seems to the writer to exist a distinction, illustrated in these cases, between that "flow" which is seen in these metals, and that to which has been attributed the relief of internal stress and the elevation of the elastic limit by strain and with time.

This last phenomenon—the exaltation of the elastic limit by strain—has been observed very strikingly, by the writer, in the deflection of iron bars, by transverse stress. The plate

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exhibits the strain-diagrams obtained by traverse deflection of 4 bars of ordinary merchant wrought-iron which were all cut from the same rod. Of these, two were tested in the machine above described, in which the deflection remains constant when the machine is untouched while the load gradually decreased—or, more properly, while the effort of the bar to regain its original form, decreases. The other two were tested by dead loads—the load remaining constant while the deflection may vary when the apparatus is left to itself.

These two pairs of specimens were broken; one in each set by adding weight steadily until the end of the test, so as to give as little time for elevation of elastic limits as was possible, and one in each set by intermittent stress, observing sets, and the elevation of the elastic limit.

If the long-known effects of cold-hammering, cold-rolling, and wire-drawing, in stiffening, strengthening, and hardening some metals can be, as the writer is inclined to believe, attributed in part to this molecular change, as well as to simple condensation and closing up of cavities and pores, this exaltation of the elastic limit by distortion under externally applied force has now been shown to occur in iron and in metals of that class in tension, torsion, compression, and under transverse strain.

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The strain-diagrams exhibited in the plate do not present to the eye one of the most important distinctions between the two classes of metals. As seen by study of these diagrams, both classes, when strained by flexure, gradually exhibit less and less effort to restore themselves to their original form.

In the case of the tin-class, this loss of straightening

power seems often to continue indefinitely, and, as in one example here illustrated, even until fracture occurs.

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With iron and the class of which that metal is typical, this reduction of effort becomes gradually less and less rapid, and finally reaches a limit after attaining which, the bar is found to have become strengthened, and the elastic limit to have become elevated. In this respect, the two classes are affected by time of strain, in precisely opposite ways.

The plate exhibits, even better than the record, the superior ultimate resistance of the bars which have been intermittently strained, as well as the elevation of the elastic limit. This parallelism of the "elasticity lines" obtained in taking sets, shows that the modulus of elasticity is unaffected by the causes of elevation of the elastic limit.

Evidence appealing directly to the senses has been presented in the course of experiment on the second class of metals, of the intra-molecular flow. When a bar of tin is bent, it emits while bending the peculiar crackling sound familiarly known as the "cry of tin." This sound has not been observed hitherto, so far as the writer is aware, when a bar has been held flexed and perfectly still. In several cases recently, in experiments on flexure of metals of the second class, bars held at a constant deflection have emitted such sounds hour after hour, while taking set and losing their power of restoration of shape.

AMERICAN SOCIETY OF CIVIL ENGINEERS.

DISCUSSION

At the Seventh Annual Convention.

FLEXURE OF BEAMS.

MR. ROBERT H. THURSTON.—Referring to "Resistances of Beams to Flexure": *

1°. I agree with Gen. Barnard in considering the formulæ of Navier, and those in common use as based upon them, as well as the arguments of Decomble sustaining the former, as not well supported by the results of experiment, except in a few special cases.

2°. The ordinary theory, and its resulting equations, in which the resistances of particles to compression and to extension are proportional to their distance from the neutral surface, are apparently sufficiently correct up to that limit of flexure at which the exterior sets of particles on the one side or on the other, are forced beyond the elastic limit.

3°. With absolutely non-ductile materials, or materials destitute of viscosity, fracture occurs at this point. But, with ordinary materials, and notably with good iron, low steel, and all of the useful metals and alloys in common employ, rupture does not then take place.

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* Resistance of Beams to Flexure, Gen. J. G. Barnard, Trans. Am. Soc. C. E., Vol. III., page 123.

4°. The exterior portions of the mass are compressed on the one side, offering more and more resistance nearly, if not quite, up to the point of actual breaking, which breaking may only occur long after passing the elastic limit. On the other side, the similar sets of particles are drawn apart, passing the elastic limit for tension, and then resisting the stress with approximately constant force, "flow" occurring until that limit of flow is reached, and rupture takes place.

5°. Fracture may occur under either of several sets of conditions.

A. The material may be absolutely brittle. (*a.*) In this case, the elastic limit and the limit of rupture coincide for both simple tension and simple compression. The piece will break with a snap when, under flexure, either limit is reached. (*b.*) Or, it may happen that the limit is reached simultaneously on both sides.

B. The material may be slightly viscous. (*a.*) The flexure of the piece will produce compression or extension, or both, beyond the elastic limit before rupture, giving three sets of conditions to be expressed by the formula. (*b.*) The increase of resistance, after passing the elastic limit, will not be similar for both forms of resistance, and each substance will probably be found characteristically distinguishable from every other. (*c.*) It would appear from experiments already familiar, that the resistance to compression will frequently increase in a very high ratio as compared with that to extension, thus swinging the neutral surface toward the compressed side, and probably sometimes approximately to the limiting surface, with very hard and friable substances, thus bringing about something like a correspondence with "Galileo's theory." This, I presume, does not often happen.

C. The material may be very ductile or viscous.* (*a.*) In this case the phenomena of flexure and rupture will be as

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* It is to be remembered that viscosity and high cohesive force may co-exist, as shown by Prof. Henry and Mon. Tresca.

last described, but of exaggerated extent and importance. (*b*.) The resistances to extension and to compression as developed in this case, will be approximately, or accurately, those observed in experiments producing rupture by direct tension and by direct compression. The neutral surface will be determined in position by the ratio of these ultimate resistances.

6°. Proposition 1, of Decomble, as rendered by Gen. Barnard,* therefore, may or may not be true for any individual case, and it cannot be true for all materials. Proposition 2 is, I think, probably correct, it being understood that the effect of "flow" in producing modification of the co-efficients of elasticity and of rupture is comprehended. Proposition 3 is, I should say, certainly incorrect for ductile substances.

Decomble is in error in claiming that Navier's theories, narrow and inflexible as are their conditions, explain "all phenomena" of flexure and rupture, or that it can always give us correct moduli of rupture, or that it is in "complete harmony" with any but a narrow range of practice.

7°. The statement that "any load, however small," is "capable of producing rupture providing that the trial is sufficiently prolonged," I have long since shown, by experiment (which has been published in this country and in Europe),† to be quite the reverse of the truth in the case of iron, steel, etc. The fact, as shown by the *fac-simile* strain-diagrams illustrating these papers, being, that, *static stress, less than that producing rupture, but greater than that corresponding with the elastic limit, produces actual increase of resisting power*. This fact has since been proven by other investigators and by quite independent methods of research.

8°. I have also shown in those experimental investigations, that the converse fact exists, that *distortion, rapidly*

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* Trans. Am. Soc. C. E., Vol. III., page 123.

† Transactions, Vol. II., page 239; Vol. III., page 12, etc.; Journal of the Franklin Institute, 1874; Van Nostrand's Engineering Magazine, 1874; London Engineering, 1873; Practical Mechanics' Magazine, 1874; *Dingler's Polytechnisches Journal*, 1875.

produced, causes an actual decrease of resisting power. Strain-diagrams were given illustrating this fact very strikingly.

9° This variation of resistance with variation of the method of rupture introduces another element of uncertainty into "Navier's theory," as well as into all formulas yet constructed. This element must remain until experiment has indicated a measure of it and the form of the function expressing its law, and thus enabled us to construct a correct formula.

10°. Referring to the remarks of Gen. Barnard which follow the paper under discussion,* we may find in the phenomena just considered a reason for the fact, remarked by him, that "beams fractured by shot did not resist any thing like so much" as those broken under the slow and steady action of the hydraulic press.

11°. The assumption that resistances vary each way from the neutral surface proportionally with their distance from that surface, is, when coupled with a rejected hypothesis of Navier, nevertheless, not far from the truth in special cases, as may be shown by proper mathematical treatment and comparison with results obtained experimentally.

12°. Mr. William Kent † made this comparison for cast and wrought-iron and for ash. The results of analysis and of experiment give the following values of the R in the ordinary formula: ‡

$$M = \tfrac{1}{6} R B D^2 \qquad (1)$$

for a beam fixed at one end, loaded at the other:

	CAST-IRON.	WROUGHT-IRON.	ASH.
R—(theoretical)	32 280	60 000	12 120
R—(experimental)	35 000	60 000	12 000

* Trans. Am. Soc. C. E., Vol. III., page 127.

† Assistant in the Department of Engineering, Stevens Institute of Technology.

‡ Wood on Resistance of Materials.

13°. This remarkable approximation is thus derived. Suppose a fixed beam with loaded extremity, the force P being a measure of the weight W, and the beam having a depth D, a breadth unity, and a neutral surface situated at a distance Y from the superior surface of the beam. Representing the resistances graphically by triangles having altitudes TN, CN; their measures in tension and compression are respectively $\frac{1}{2}\,TN$, $\frac{1}{2}\,CN$, and their moments are $\frac{1}{2}\,TN \times \frac{2}{3}\,Y = \frac{1}{3}\,TY^2$, and $\frac{1}{2}\,CN \times \frac{2}{3}\,(D-Y) = \frac{1}{3}\,C\,(D-Y)^2$. An early hypothesis of Navier, which seems to have been entirely abandoned by him subsequently, and which has not been accepted by subsequent writers on the subject, makes these moments equal. Assuming this to be correct,

$$TY^2 = C\,(D-Y)^2 \qquad (2)$$

$$\text{and } \frac{C}{T} = \frac{Y^2}{(D-Y)^2} \qquad (3)$$

and, from this expression, we may find the position of the neutral surface, as determined by the assumed conditions. Then, letting B = the breadth of the beam,

$$WL = \tfrac{1}{3}\,B\,[TY^2 + C\,(D-Y)^2] = \tfrac{1}{6}\,RBD^2 \qquad (4)$$

in which latter expression R is the modulus of rupture, and its value can be found when C and T are known. It will always be of a value intermediate between T and C.

14°. The following are the data and results for the three cases taken: the results are well worthy of examination and record.

	T	C	B	D	L	Y	D-Y	WL	R
Cast-iron	16 000	96 000	1	1	1	0.71	0.29	5 380	32 280
Wrought-iron	60 000	60 000	1	1	1	0.5	0 5	10 000	60 000
Ash timber	17 200	9 000	1	1	1	0.42	0.58	2 020	12 120

15°. The common theory of rupture, as it is defined by Prof. Wood, is confessedly far from correct, and, as shown at the beginning of these remarks, the neutral surface must vary in position, and cannot invariably pass through the centre of gravity of section. It would seem that such coincidence of position, when occurring at all, is simply a matter of incidental concurrence of conditions.

16°. The accurate mathematical expression of the phenomena of flexure and rupture, as already remarked, must be vastly more comprehensive and flexible, and more facile of application than any yet proposed. As I have shown, it is not sufficient that both R_1 and R_2—that is, both T and C—appear in the formulas, as proposed by Decomble. The real value of these quantities, as there appearing, must vary as some function of distance from the neutral line, while the position of the neutral line must itself vary with both the value of T and C in different cases, and with their change of value in the same beam, as flexure progresses and after rupture commences. These variable functions must all be taken into account and comprised in the general expression for the moment resisting fracture.

The characters of these functions, however, are unfortunately not yet ascertained, and it is only after experiments in which the moment of resistance is accurately measured during every stage of fracture, and so completely that the strain-diagram of the experiments can be graphically given, or its equation constructed, that we can obtain their values. This has, as yet, been done in but few cases, as in some experiments of Hodgkinson, in the work of Styffe, and in experiments made by Rodman.

17°. It does not necessarily follow that the formulas finally resulting must be either complex or inconvenient of application. Simple expressions will, at least, be found for special cases of simple character, which will serve every purpose of the engineer.

18°. It is also true, as stated by Prof. Wood, and as known

by every engineer, that the phenomena of flexure within admissible limits are much less difficult of manipulation than those of rupture, or than those resulting in serious permanent distortion. Hence, it is true that ordinary engineering practice is not placed at such a serious disadvantage as these defects of the theory of strain might seem to indicate.

19°. In common with every member of the profession, I am called upon to admit the great services rendered us by Navier in the splendid work done by him at *l'École des Ponts et Chaussées*, in establishing a theory of engineering, as well as in working up a theory of rupture, and I desire to acknowledge those services, while declining to admit absolute accuracy in his theories. They were constructed at a time when science was apparently divorced from the practice of engineering, and when his services in securing a genuine union were most invaluable. He must always be regarded as one of the great leaders in our profession.

I would unite with Gen. Barnard in his remarks, relative to the attempt of Navier's pupil, Decomble, to retain the theory while modifying the formulas of Navier: "If by discarding a coefficient founded upon an imaginary coefficient of elasticity, and the introduction of distinct and independent factors, symbolic of resistance to rupture by compression and extension, it is shown that the Navier formula can be made reliable, an important service has been rendered to engineering science." *

I would myself add that the discovery and the mathematical expression of the varying functions which I have described, and the establishment of formulas of application embodying the facts of the variation of the coefficient of elasticity, of that of the module of resistance of rupture by tension and compression, and in the position of the neutral

* Vol. III., Trans. Am. Soc. C. E., page 126.

surface which are still, as previously, essential, but unknown elements of a correct theory of strain; all of these yet remain to compensate some skilful experimenter and expert analyst. Their determination would earn for their fortunate discoverer higher distinction than ever won by either Coulomb or Navier.

REDUCTION TABLES.

TABLE I.

FOR CONVERTING METRES INTO INCHES.

METRES	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
0	0	39·4	78·7	118·1	157·5	196·8	236·2	275·6	315·0	354·3
10	393·7	433·1	472·4	511·8	551·2	590·5	629·9	669·3	708·7	748·0
20	787·4	826·8	866·1	905·5	944·9	984·2	1023·6	1063·0	1102·4	1141·7
30	1181·1	1220·5	1259·8	1299·2	1338·6	1377·9	1417·3	1456·7	1496·1	1535·4
40	1574·8	1614·2	1653·5	1692·9	1732·3	1771·6	1811·0	1850·4	1889·8	1929·1
50	1968·5	2007·9	2047·2	2086·6	2126·0	2165·3	2204·7	2244·1	2283·5	2322·8
60	2362·2	2401·6	2440·9	2480·3	2519·7	2559·0	2598·4	2637·8	2677·2	2716·5
70	2755·9	2795·3	2834·6	2874·0	2913·4	2952·7	2992·1	3031·5	3070·9	3110·2
80	3149·6	3189·0	3228·3	3267·7	3307·1	3346·4	3385·8	3425·2	3464·6	3303·9
90	3543·3	3582·7	3622·0	3661·4	3700·8	3740·1	3779·5	3818·9	3858·3	3897·6

In general, where great accuracy is not required, we need only multiply the length in metres by 3·28, and the product will be the equivalent length *in feet.*

EXAMPLE.—57 metres = 1968·5 inches = 50 metres, plus
275·6 " = 7 " or

2244·1 " = 57 "

570 metres = 19685 inches = 500 metres, plus
2756 " = 70 " or

22441 " = 570 "

22441 inches = 1870 feet. 570 × 3·28 = 1869·6 feet.

TABLE II.

FOR CONVERTING SQUARE CENTIMETRES INTO SQUARE INCHES.

Square Centimetres.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
0	0	0·155	0·310	0·465	0·620	0·775	0·930	1·085	1·240	1·395
10	1·55	1·705	1·860	2·015	2·170	2·325	2·480	2·635	2·790	2·945
20	3·10	3·255	3·410	3·565	3·720	3·875	4·030	4·185	4·340	4·495
30	4·65	4·805	4·960	5·115	5·270	5·425	5·580	5·735	5·890	6·045
40	6·20	6·355	6·510	6·665	6·820	6·975	7·130	7·285	7·440	7·595
50	7·75	7·905	8·060	8·215	8·370	8·525	8·680	8·835	8·990	9·145
60	9·30	9·455	9·610	9·765	9·920	10·075	10·230	10·385	10·540	10·695
70	10·85	11·005	11·160	11·315	11·470	11·625	11·780	11·935	12·090	12·245
80	12·40	12·555	12·710	12·865	13·020	13·175	13·330	13·485	13·640	13·795
90	13·95	14·105	14·260	14·415	14·570	14·725	14·880	15·035	15·190	15·345

TABLE III.

FOR CONVERTING KILOGRAMS INTO AVOIRDUPOIS POUNDS

Kilograms.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
0	0	2·205	4·409	6·614	8·818	11·023	13·228	15·432	17·637	19·841
10	22·046	24·251	26·455	28·660	30·864	33·069	35·274	37·478	39·683	41·887
20	44·092	46·297	48·501	50·706	52·910	55·115	57·320	59·524	61·729	63·933
30	66·138	68·343	70·547	72·752	74·956	77·161	79·366	81·570	83·775	85·979
40	88·184	90·389	92·593	94·798	97·002	99·207	101·412	103·616	105·821	108·025
50	110·230	112·435	114·639	116·844	119·048	121·253	123·458	125·662	127·867	130·071
60	132·276	134·481	136·685	138·890	141·094	143·299	145·504	147·708	149·913	152·117
70	154·322	156·527	158·731	160·936	163·140	165·345	167·550	169·754	171·959	174·163
80	176·368	178·573	180·777	182·982	185·186	187·391	189·596	191·800	194·005	196·209
90	198·414	200·619	202·823	205·028	207·232	209·437	211·642	213·846	216·051	218·255

TABLE IV.

FOR CONVERTING KILOGRAMS PER LINEAL METRE INTO POUNDS PER LINEAL FOOT.

Kilo-grams per Metre.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
0	0	0·672	1·344	2·016	2·688	3·360	4·032	4·704	5·376	6·048
10	6·720	7·392	8·064	8·736	9·408	10·080	10·752	11·424	12·096	12·768
20	13·440	14·112	14·784	15·456	16·128	16·800	17·472	18·144	18·816	19·488
30	20·160	20·832	21·504	22·176	22·848	23·520	24·192	24·864	25·536	26·208
40	26·880	27·552	28·224	28·896	29·568	30·240	30·912	31·584	32·256	32·928
50	33·600	34·272	34·944	35·616	36·288	36·960	37·632	38·304	38·976	39·648
60	40·320	40·992	41·664	42·336	43·008	43·680	44·352	45·024	45·696	46·368
70	47·040	47·712	48·384	49·056	49·728	50·400	51·072	51·744	52·416	53·088
80	53·760	54·432	55·104	55·776	56·448	57·120	57·792	58·464	59·136	59·808
90	60·480	61·152	61·824	62·496	63·168	63·840	64·512	65·184	65·856	66·528

In general, where great accuracy is not required, $\frac{2}{3}$ of the number of kilograms per metre will give the equivalent number of pounds per foot.

TABLE V.

FOR CONVERTING KILOGRAMS PER SQUARE CENTIMETRE INTO POUNDS PER SQUARE INCH.

Kilograms per Square Centimetre.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
0	0	14·223	28·446	42·669	56·892	71·115	85·338	99·561	113·784	128·007
10	142·230	156·453	170·676	184·899	199·122	213·345	227·568	241·791	256·014	270·237
20	284·460	298·683	312·906	327·129	341·352	355·575	369·798	384·021	398·244	412·467
30	426·690	440·913	455·136	469·359	483·582	497·805	512·028	526·251	540·474	554·697
40	568·920	583·143	597·366	611·589	625·812	640·035	654·258	668·481	682·704	696·927
50	711·150	725·373	739·596	753·819	768·042	782·265	796·488	810·711	824·934	839·157
60	853·380	867·603	881·826	896·049	910·272	924·495	938·718	952·941	967·164	981·387
70	995·610	1009·833	1024·056	1038·279	1052·502	1066·725	1080·948	1095·171	1109·394	1123·617
80	1137·840	1152·063	1166·286	1180·509	1194·732	1208·955	1223·178	1237·401	1251·624	1265·847
90	1280·070	1294·293	1308·516	1322·739	1336·962	1351·185	1365·408	1379·631	1393·854	1408·077

In general, where great accuracy is not required, multiply the number of kilograms per square centimetre by 100 and divide the product by 7, and the result will be the number of pounds per square inch.

TO REDUCE CENTIGRADE DEGREES TO FAHRENHEIT.

$$\tfrac{9}{5}\,C + 32 = F,$$

where C is the number of degrees centigrade, inserted with its proper sign plus or minus, and F is the equivalent number of degrees Fahrenheit.

Fig.